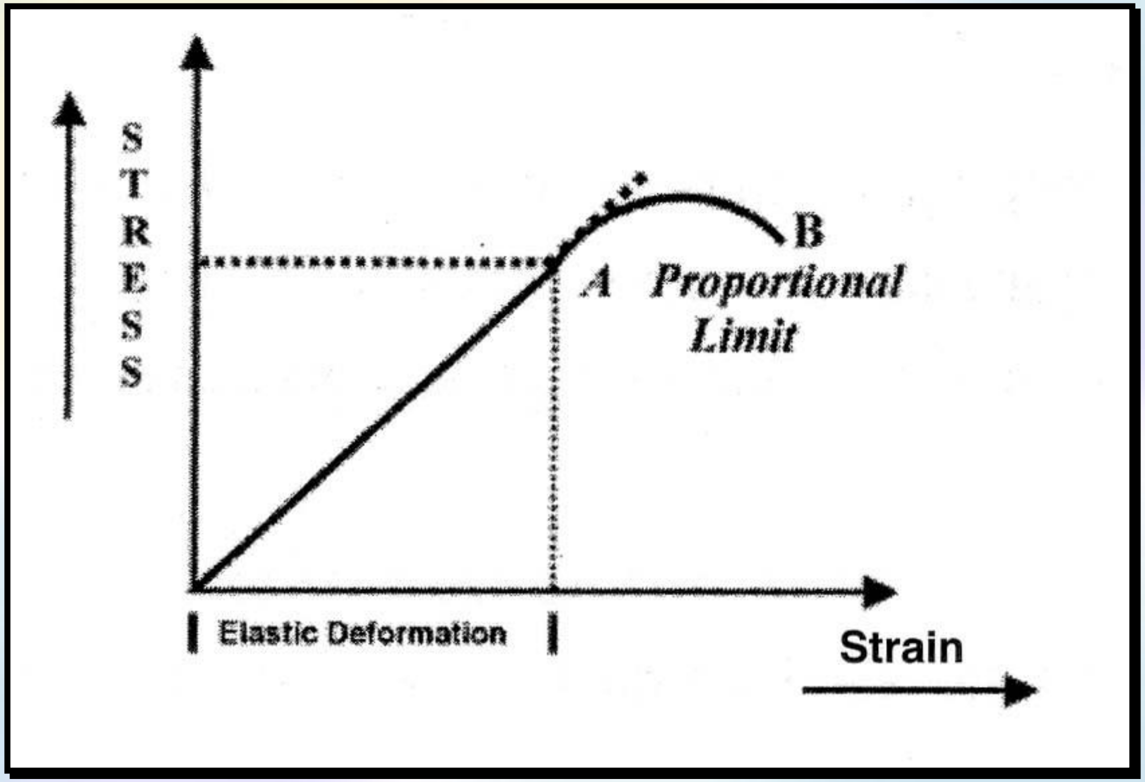
Elasticity- Energy

Proportional limit or elastic limit

The proportional limit is defined as the maximum stress that a material will sustain without the deviation of Proportionality of stress to strain.

- **Below the proportional limit no permanent deformation occurs in a structure.**
- **The region or area of the stress-strain curve below the proportional limit is called the Elastic region.**

- **The region beyond the proportional limit is called the plastic region.**
- **The elastic limit is defined as the maximum stress that a material will sustain without permanent deformation.**



Stress- strain curve for a material subjected to a tensile stress

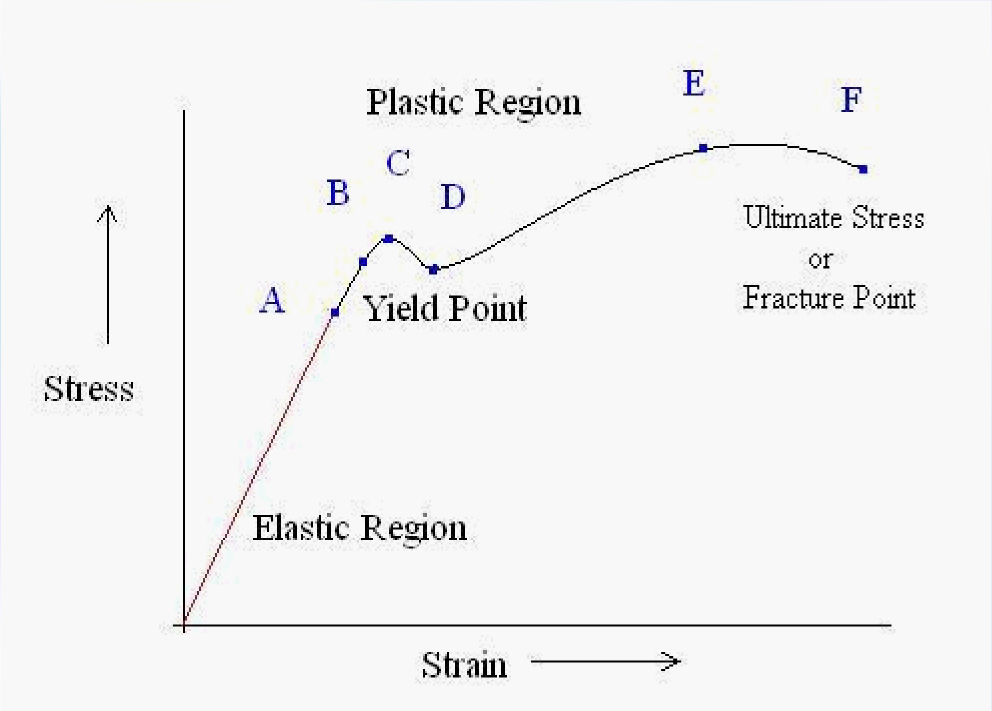
Yield strength

The yield strength or yield stress (YS) is the stress at which the material begins to function in a plastic manner. At this stress a limited permanent deformation has occurred in the material.

The yield strength is defined as the stress at which a material exhibits a specified limiting deviation of proportionality of stress to strain.

Ultimate strength

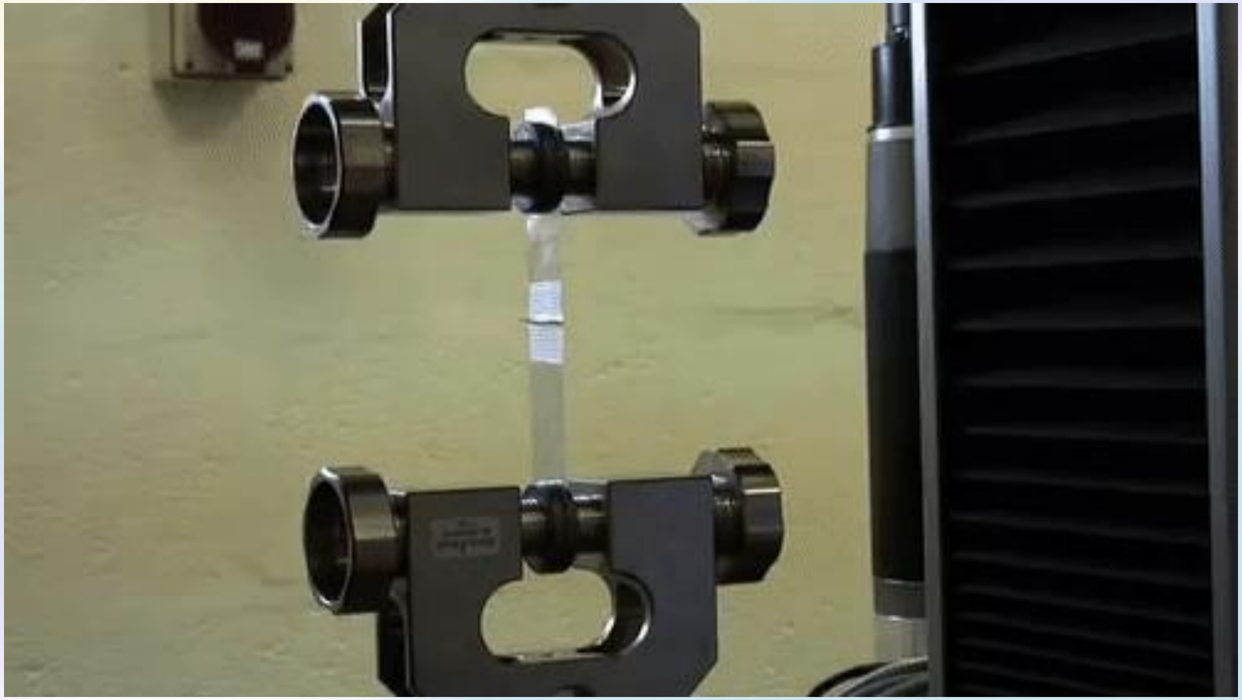
- **The ultimate tensile strength is the maximum stress that a material can withstand before failure in tension.**



In restorative dentistry, the yield strength is more important than ultimate strength because it indicates when a material starts to permanently deform.

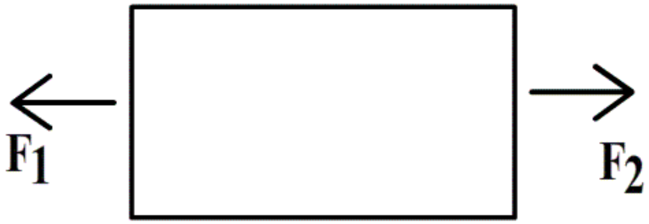
Fracture strength

The stress at which a material fracture is called fracture strength.



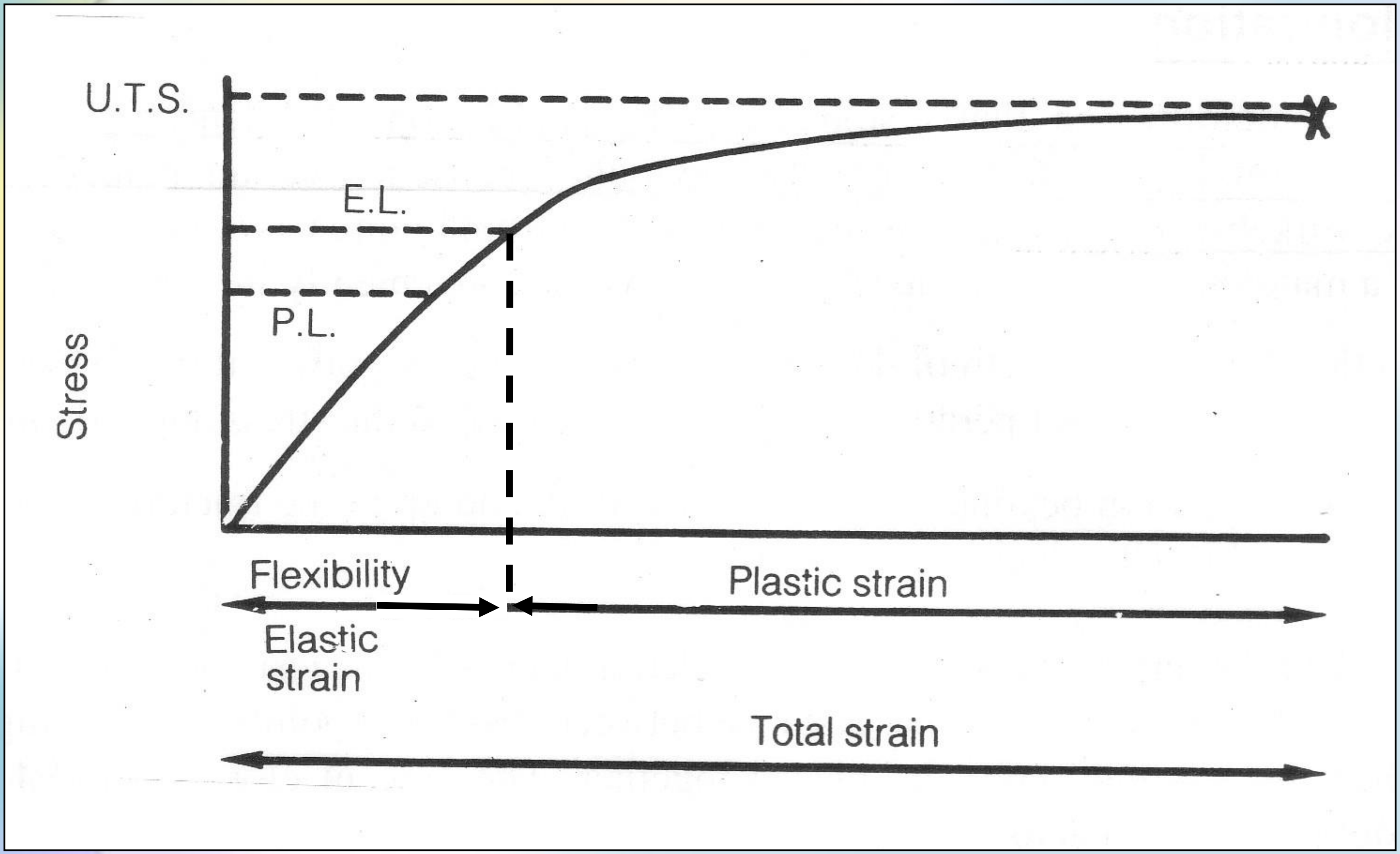
Elongation

- **The deformation that results from the application of tensile stress is elongation.**
- **Elongation is important because it gives an idea about the workability of the alloy.**



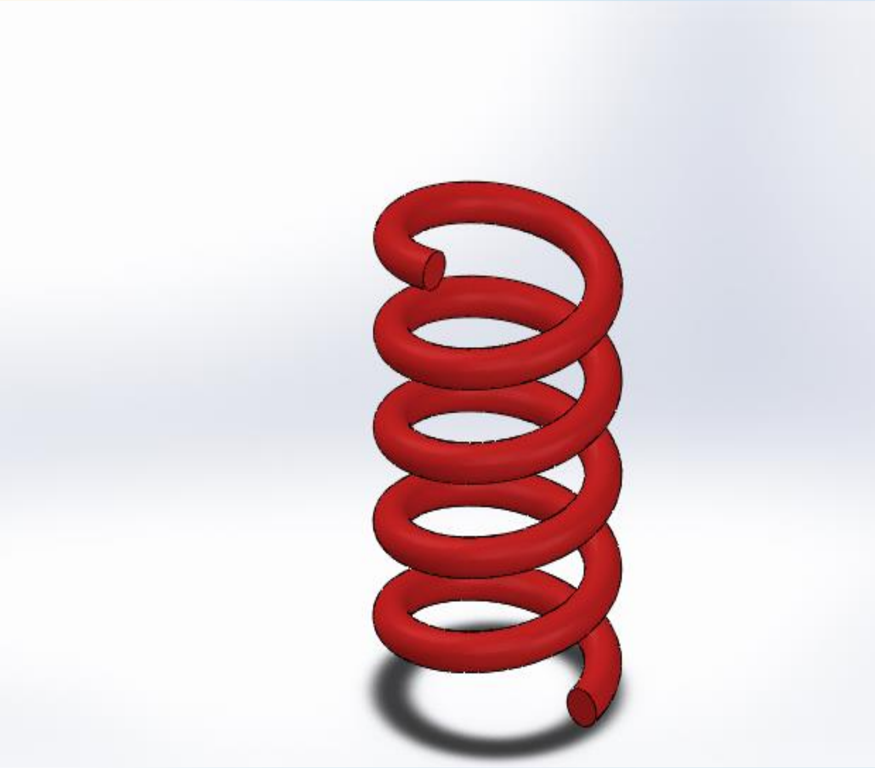
- **% of Elongation =** $$\frac{Increase\,in\,length}{Original\,length}$$ **X 100**
- **An alloy with high percent of elongation can be bent or adjusted without danger of fracture.**

Total strains



Flexibility

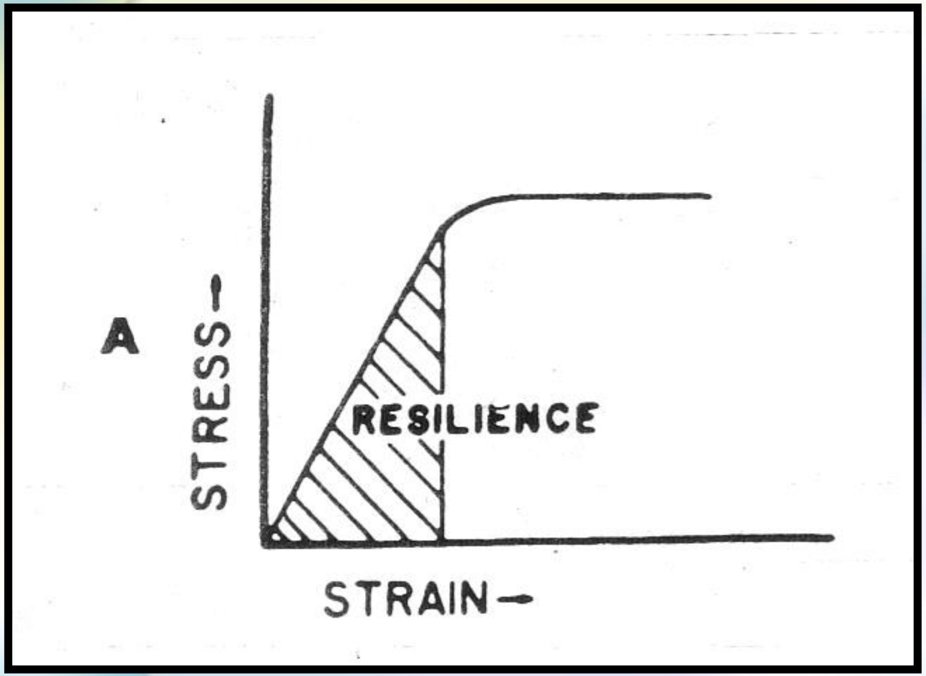
- The term flexibility describes the amount of strain up to the elastic limit.
- So that flexibility is the total amount of elastic strain in a material.



Resilience

- **Resilience is the resistance of a material to permanent deformation.**
- **It indicates the amount of energy required to deform a material to its proportional limits.**
- **Resilience is measured as the triangular area under the elastic portion of stress-strain curve. The surface area of the triangle is (*1/2* base*height*).**

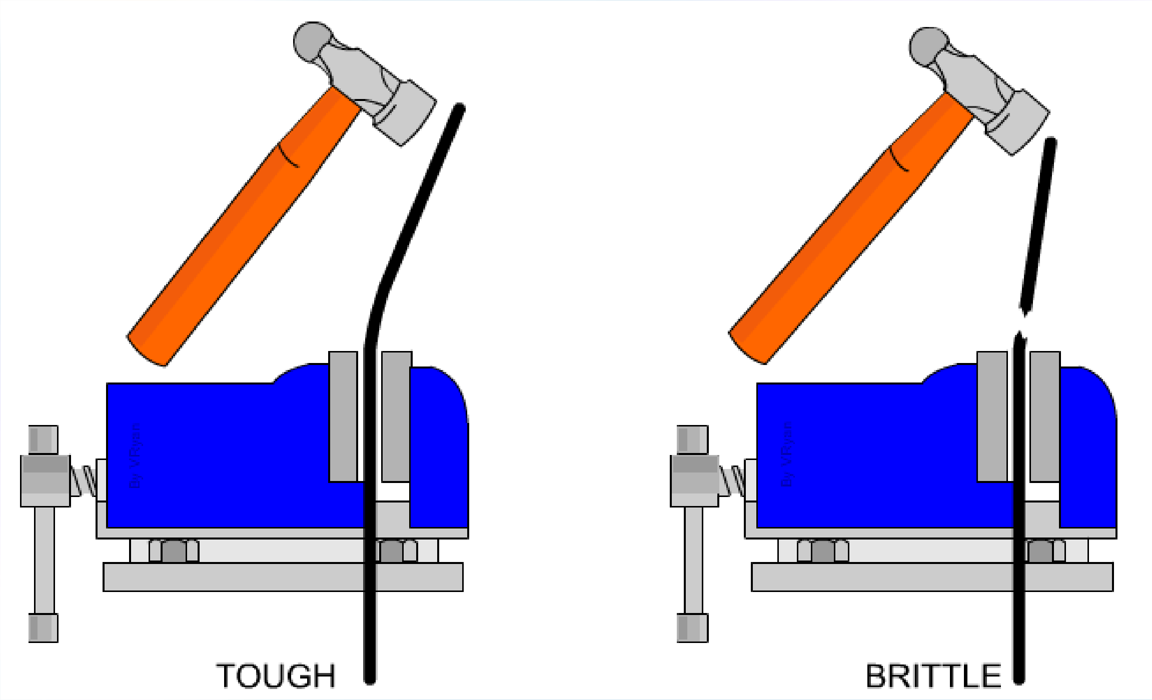
Stress-strain curves showing the area representing the resilience of a material



- **The units are represent the energy per unit volume of the material.**
- **This property is important for orthodontic wire and denture liners.**

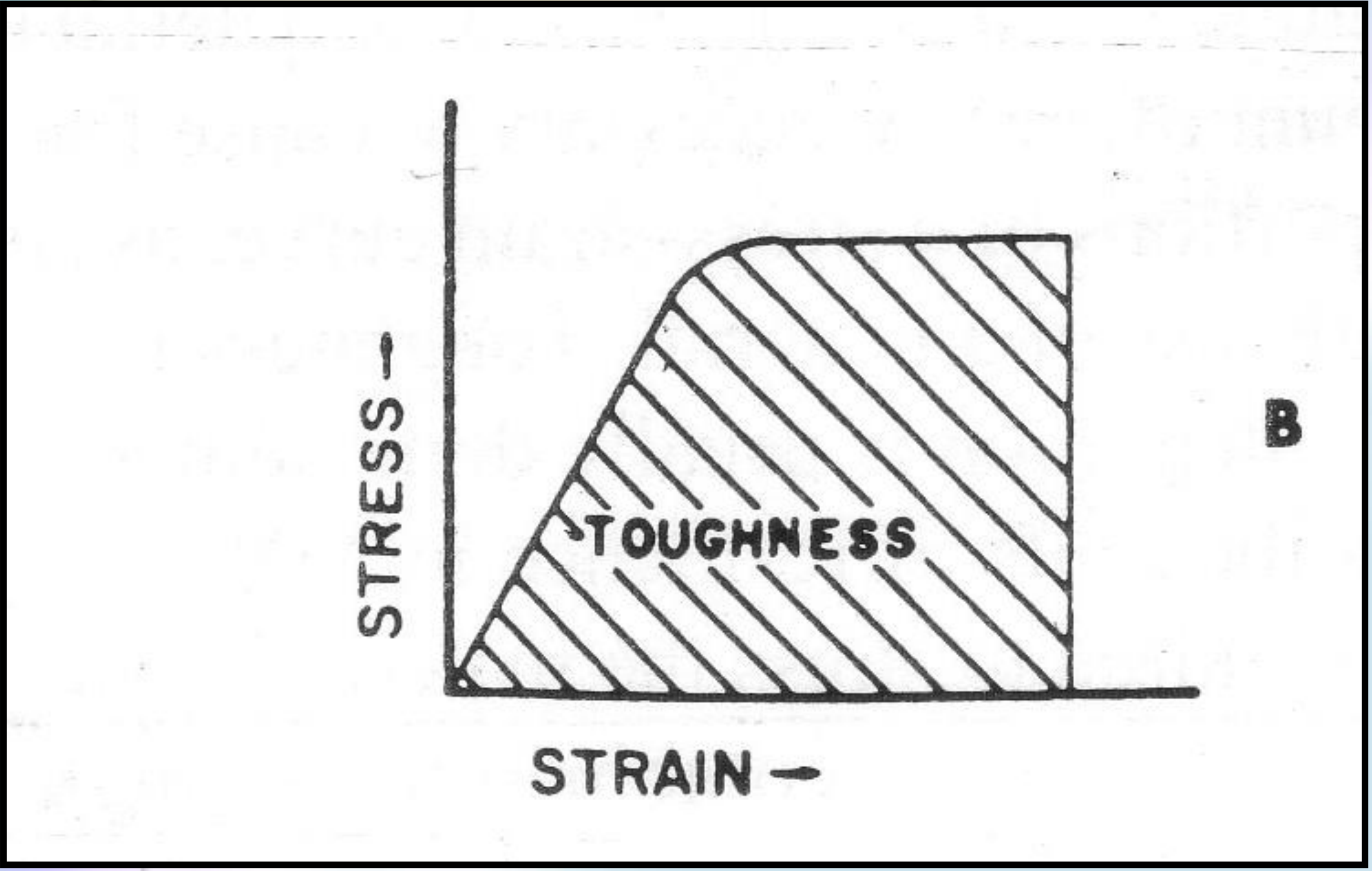
Toughness

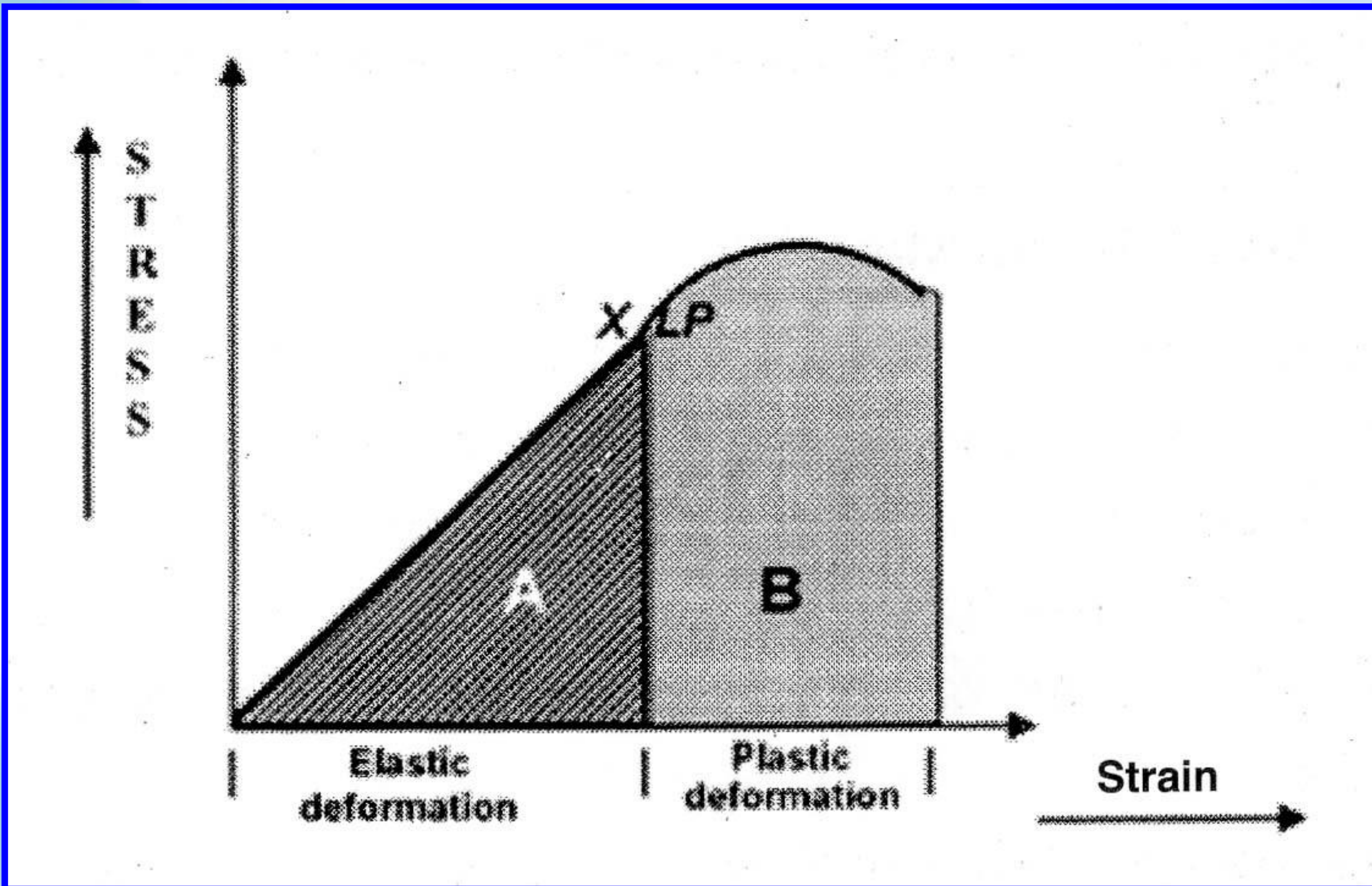
- **Toughness is defined as the amount of energy required to stress a material to the point of fracture.**
- **Toughness is the resistance of a material to fracture.**



- **The area under the elastic and plastic portion of the curve represents the toughness of a material.**
- **It is difficult to calculate and the units of toughness are the same as resilience .**

Stress-strain curves showing the area indicating the toughness.



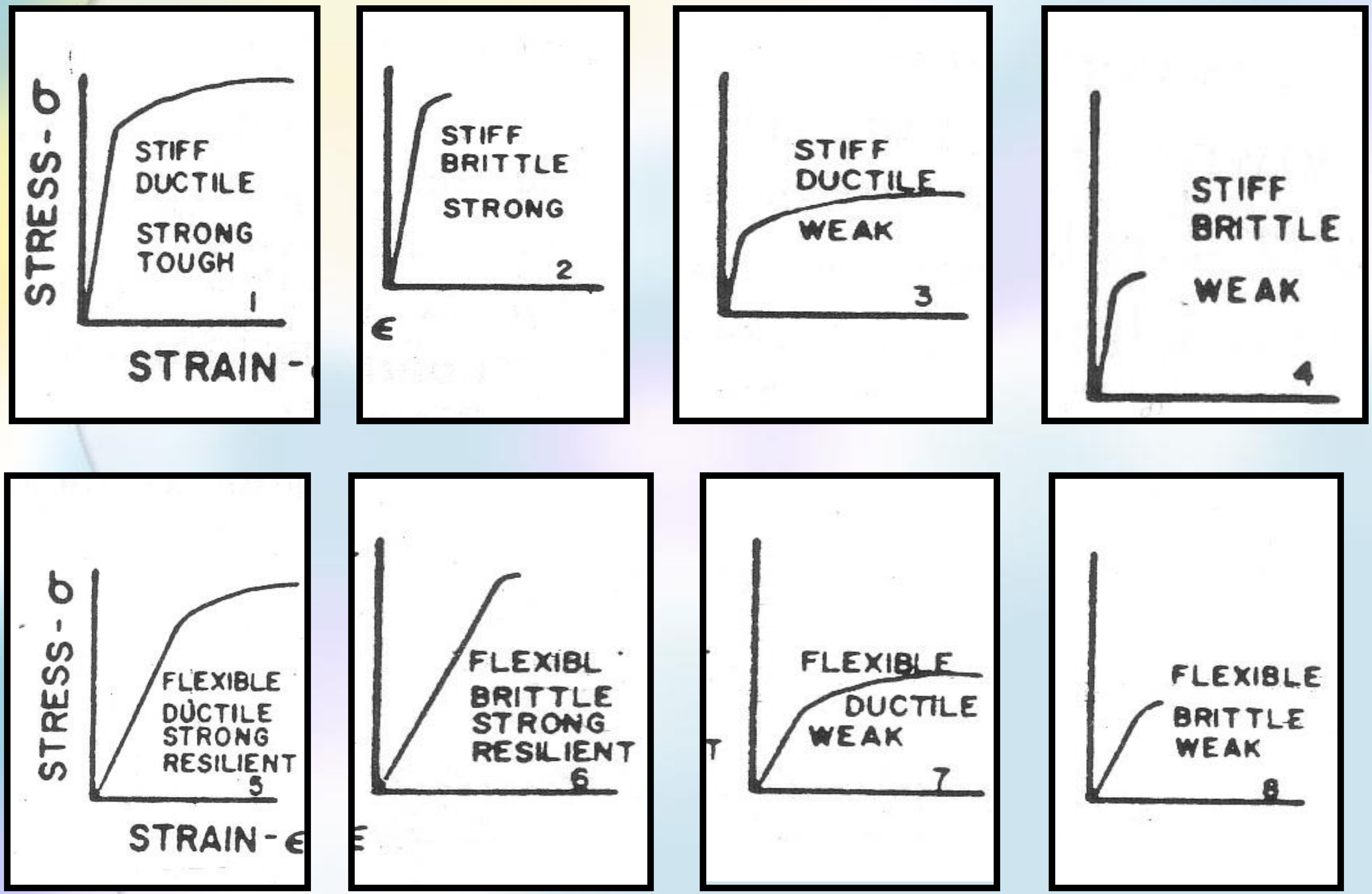


STRESS
X
LP
A
B
Elastic deformation
Plastic deformation
Strain

Properties and Stress-Strain Curves

The shape of a stress-strain curve and the magnitude of stress and strain allows the classification of materials as regard to their properties e.g. rigid, strong, solid, weak and brittle.

Stress-strain curves for materials with various combinations of properties

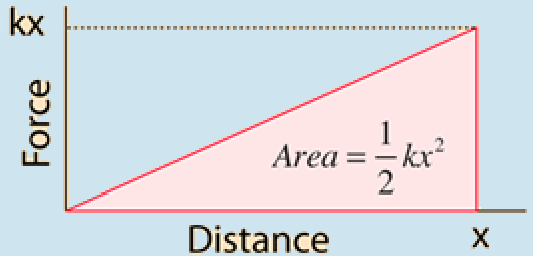


How to predict the properties of a material under testing from the stress-strain curve?

1. **The magnitude of the curve.**
2. **The inclination of the curve in the elastic range.**
3. **The amount of plastic deformation.**

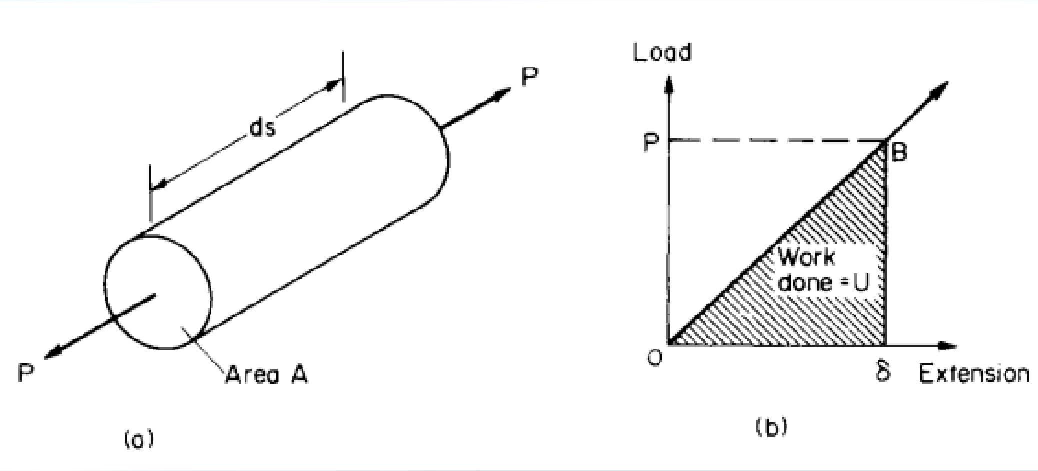
STRAIN ENERGY (Potential)

- In its widest sense the potential energy of an object is energy that is attributed to it by virtue of its position (or shape), as opposed to kinetic energy which corresponds to motion. The energy associated with change of shape such as the stretching of a spring is called strain energy.
- Strain energy is as the energy which is stored within a material when work has been done on the material. Here it is assumed that the material remains elastic whilst work is done on it so that all the energy is recoverable and no permanent deformation occurs due to yielding of the material,
- Strain energy U = work done
- Thus for a gradually applied load the work done in straining the material will be given by the shaded area under the load-extension graph of Fig.



Strain energy - Tension or compression

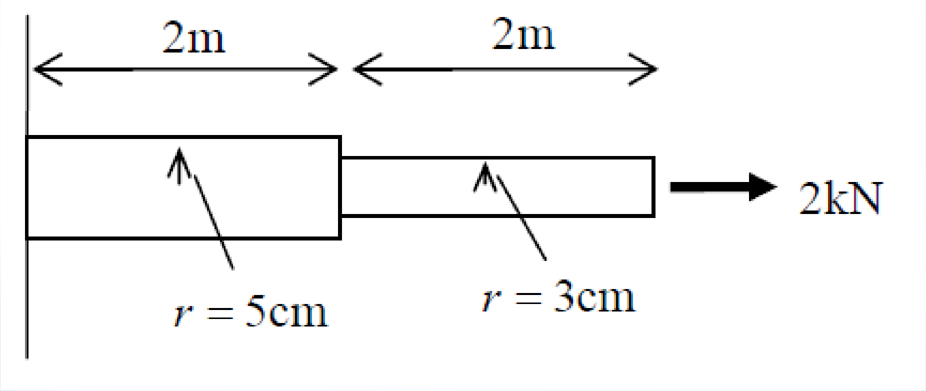
- Consider a small element of a bar, length ds, shown in Fig. If a graph is drawn of load against elastic extension the shaded area under the graph gives the work done and hence the strain energy
- $U = \frac{P^2}{2E} \times Volume$



Where ;
P= tensile force
The positive sign being used when P is tensile and the negative sign when P is compressive
E = young modulus
V= (L, length) X (A, area)

Example-2

- Consider the bar with varying circular cross-section shown in Fig. The Young's modulus is 200GPa .



- The strain energy stored in the bar when a force of 2kN is applied at the free end is;

- $U = \frac{P^2}{2E} \times Volume = U = \frac{P^2}{2E} \times A \times L = \frac{2^4}{2 \times 2 \times 10^{11}} 2 \times (\pi 5^2 + \pi 3^2) =$

Example-3

- A rod of area 90 mm^2 has a length of 3 m. Determine the strain energy if a stress of 300 MPa is applied when stretched. Young's modulus is given as 200 GPa.
- **Solution:**

Given:

Area A = 90 mm^2,

Length l = 3m,

Stress P = 300 MPa,

Young's modulus E = 200 GPa.

Volume V is given by the formula,

$V = AL = (90 \times 10^{-6}) \times 3 = V = 27 \times 10^{-6}\ m^3$

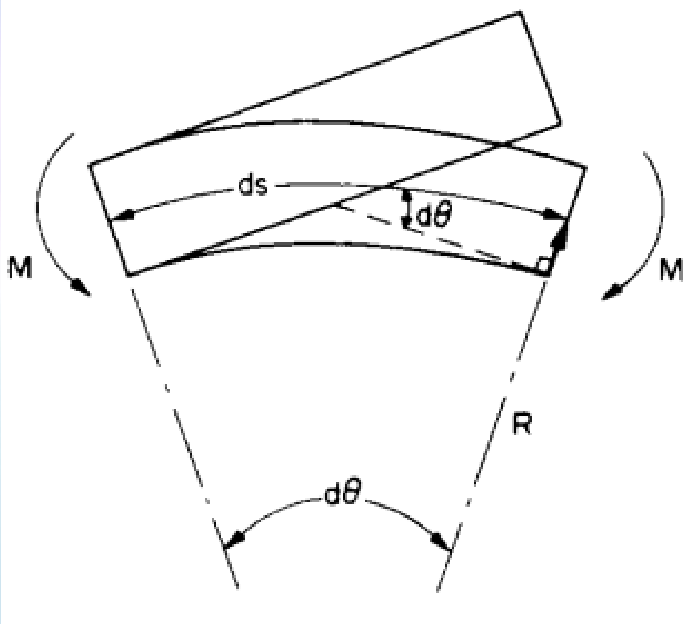
- The strain energy formula is given as,

$U = (P^2 / 2E) \times V = (300 \times 10^6)^2 / 2 \times 200 \times 10^9 \times 27 \times 10^{-6} = 12.15$ J.

- Therefore, the strain energy of rod is **12.15 J.**

Strain energy – Bending

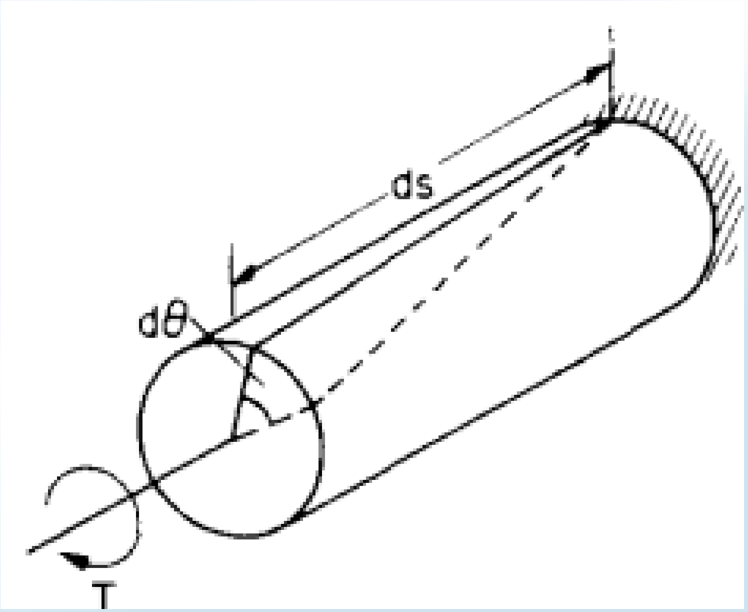
- Let the element now be subjected to a constant bending moment **M** causing it to bend into an arc of radius **R** and subtending an angle **dϴ** at the center (Fig). The beam will also have moved through an angle **dϴ**
- $U = \frac{M^2 L}{2EI}$



Where ;
EI = bending stiffness
M = bending moment
L= length

Strain energy – Torsion

- The element is now considered subjected to a torque T as shown in Fig., producing an angle of twist dϴ radians
- $U = \frac{Q^2}{4G} \times Volume$

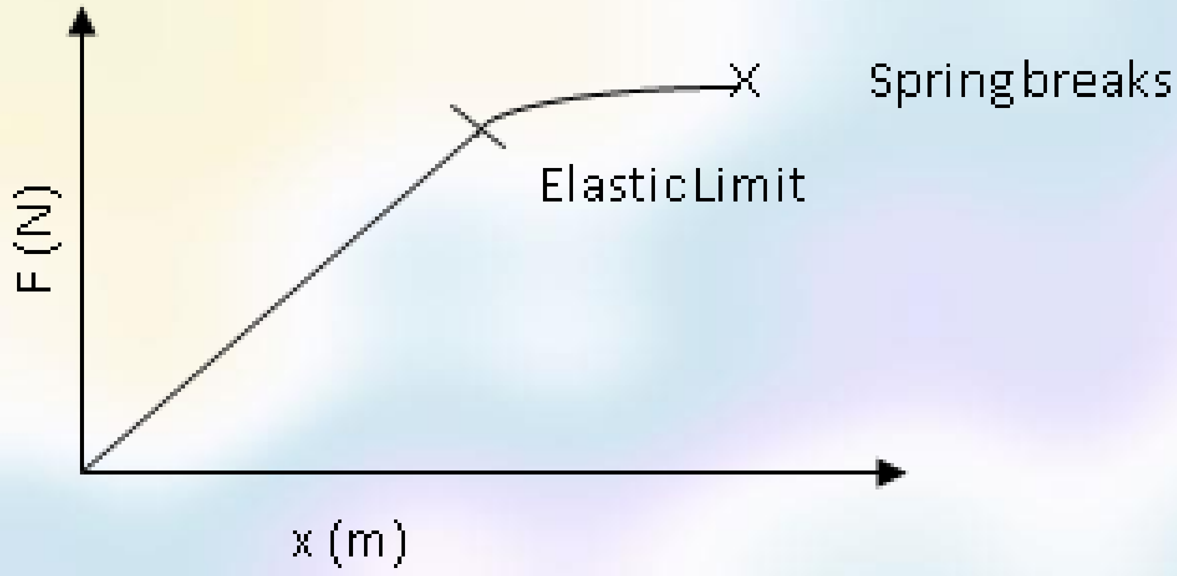


Where ;
Q= torque
G = modulus of rigidity
V= (L, length) x (A, area)

Elastic Potential Energy of Spring

- Elastic potential energy is stored in an object if there is no net deformation.
- This means that the object can return to its original form.
- Work put into extending a spring, for example, is equal to the work released by the spring when it returns to its natural shape. (Some or little heat results.)

Elastic Potential Energy of Spring



- F = kx in the elastic region only.
- The slope is the spring constant k that varies with material. (N/m)

Elastic Potential Energy of Spring

- The energy stored in a spring is equal to the work done to displace the spring which is represented by the area under the graph for the elastic region.
- Area = (1/2) base height = (1/2) k x. x = (1/2) k x^2
- E_p = (1/2) k x^2 (again measured in Joules)

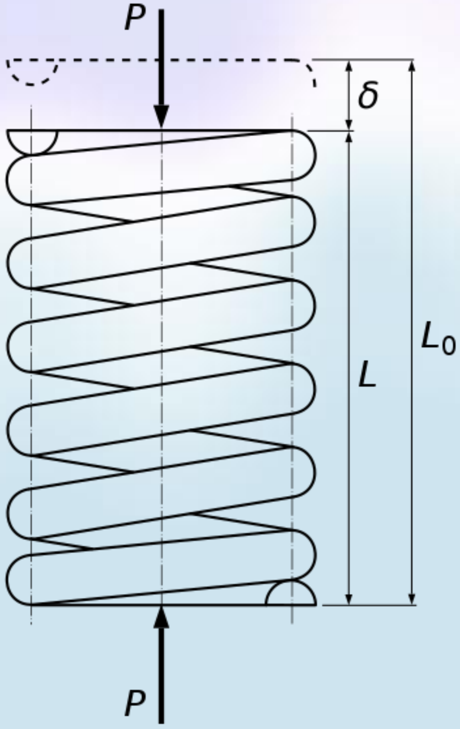
Example 4

- Find the elastic potential energy of a spring (k = 160 N/m) compressed 8 cm.
- Lo- L= 8 cm

$$E_p = (1/2)\,kx^2$$

$$= (1/2)\,(160\ \text{N/m})\,(-0.080\ \text{m})^2$$

$$= 0.51\ \text{J}$$



Example 5

- A 0.5 kg hockey puck slides at 15 m/s. It hits a spring loaded bumper (k = 360 N/m); how much does the bumper spring compress at maximum compression?
- The puck loses E_k to do work to compress the spring;

$$- \Delta E_k = \Delta E_p$$

$$-(0 - (1/2)\, mv^2) = (1/2)\, kx^2 - 0$$

$x = \pm\, 0.56$ m   As compressed, $x = -0.56$ m