Properties of solids

Elastic Stress-Strain Relationships

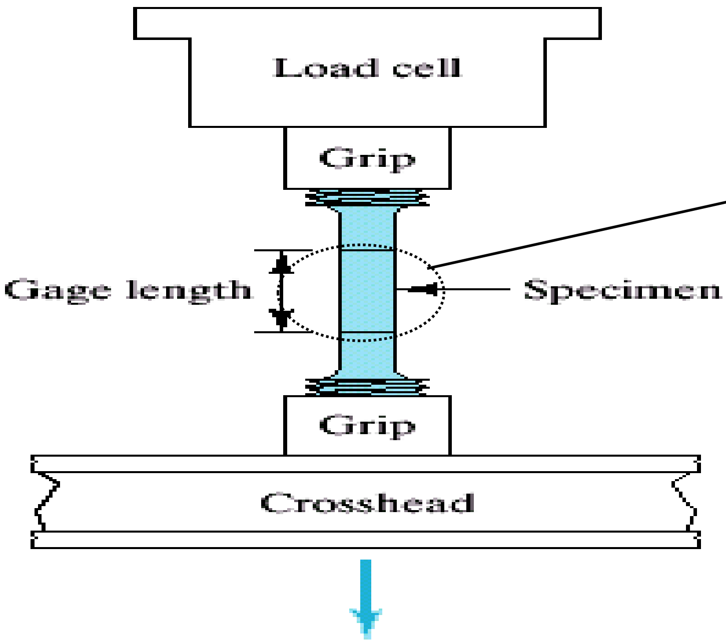
Properties of Metals

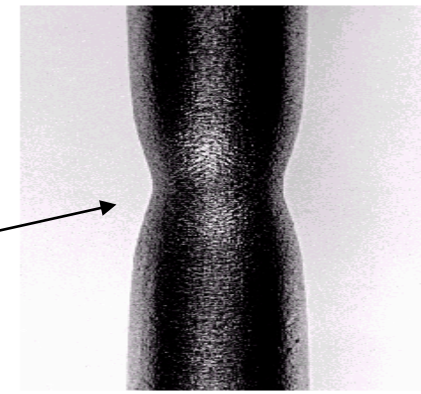
How do metals respond to external loads?

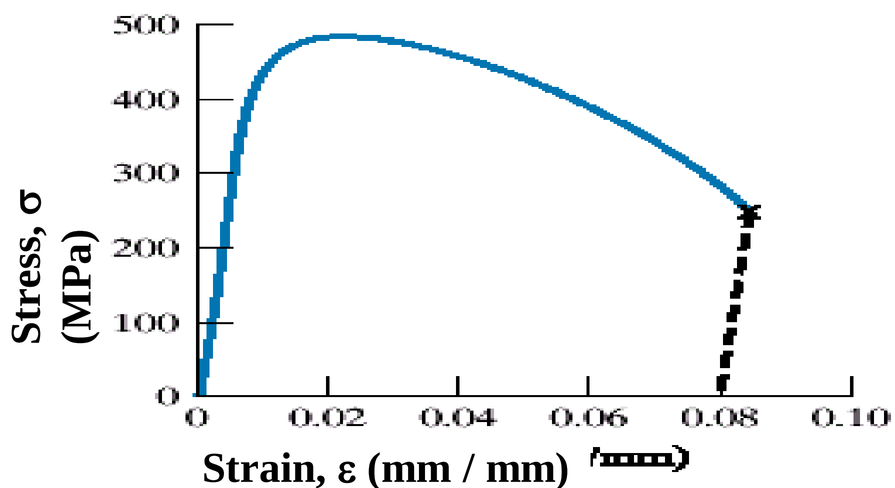
- **Stress and Strain**
  - **Tension**
  - **Compression**
  - **Shear**
  - **Torsion**
- **Elastic deformation**
- **Plastic Deformation**

Introduction

How materials deform as a function of applied load →
Testing methods and language for mechanical properties of materials.





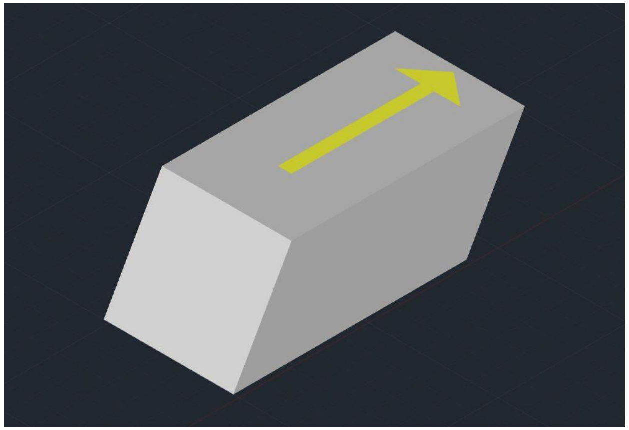


Types of Loading

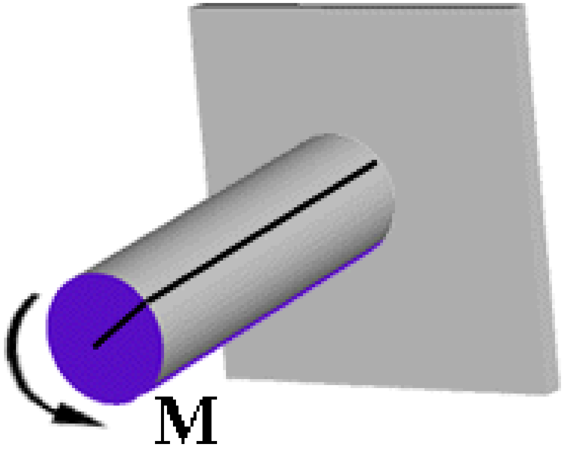
Tensile

Compressive

Shear



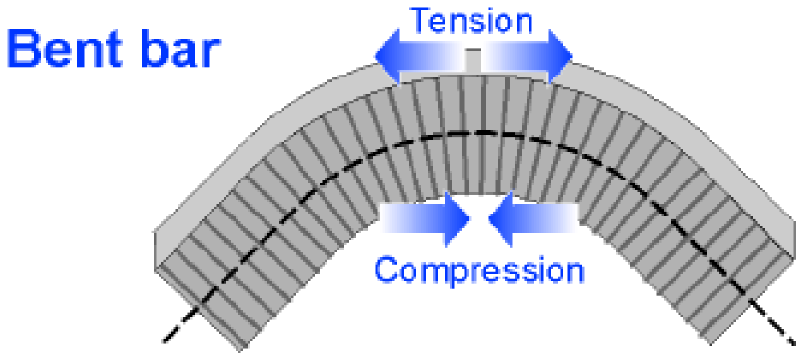
Torsion



Tensile strength

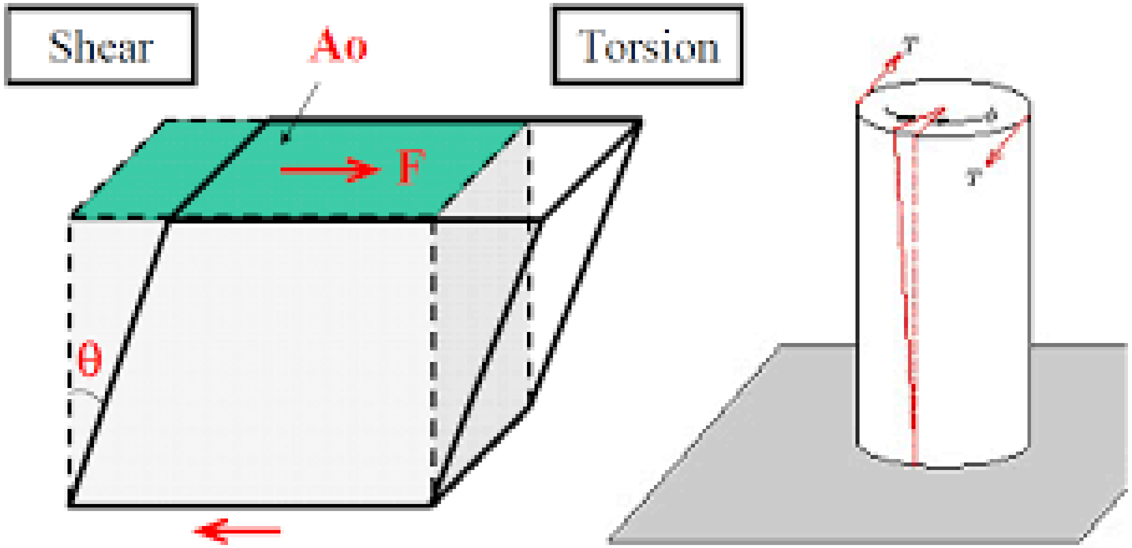
- The tensile strength is the stress at which a material breaks under a tension force.

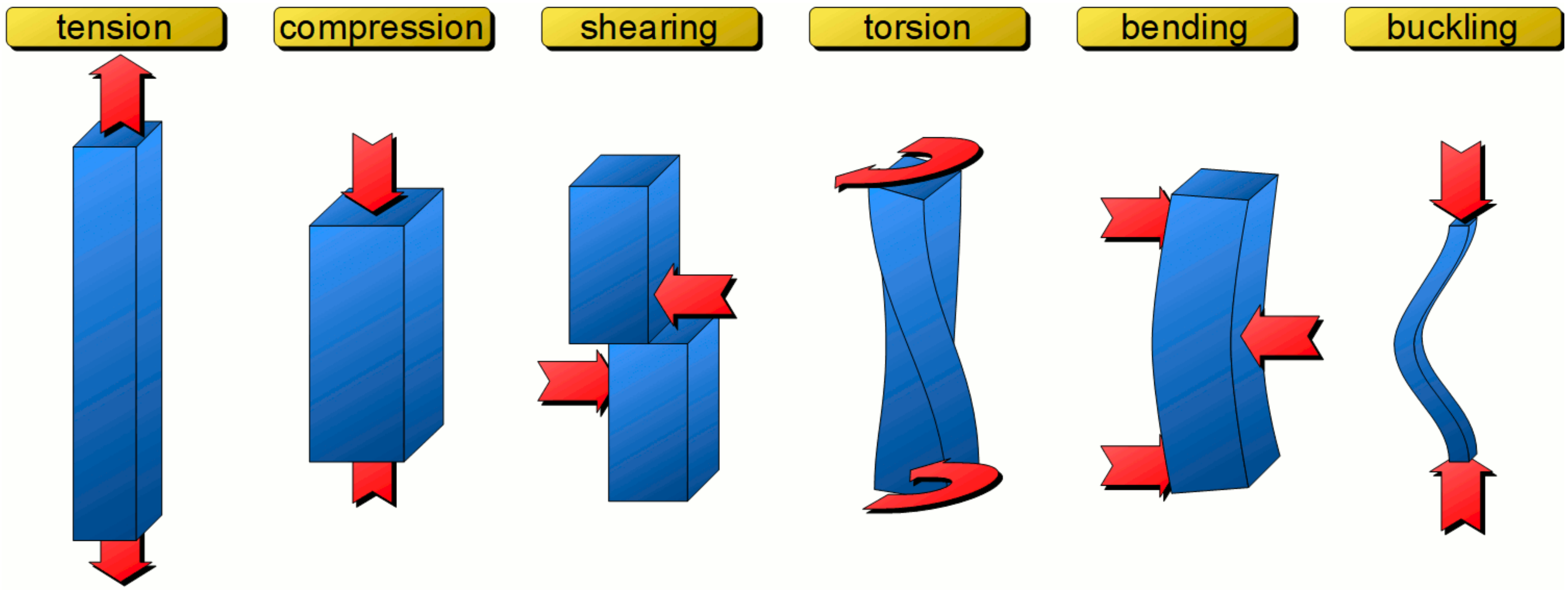
- The tensile strength also describes how materials break in bending.

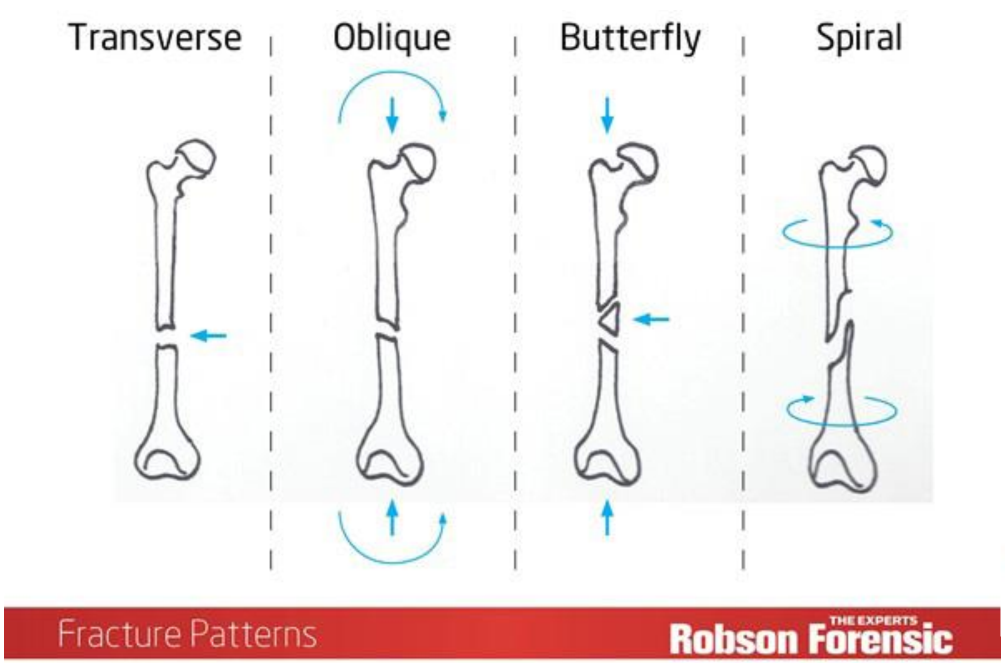


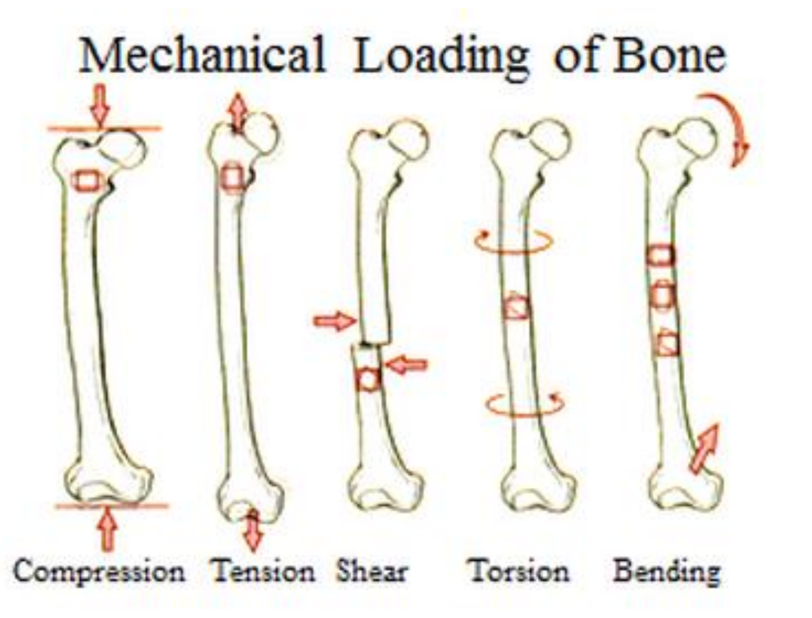
Torsion strength

Measure of the ability of a material to withstand a twisting load. It is the ultimate strength of a material subjected to torsional loading, and is the maximum torsional stress that a material sustains before rupture




tension
compression
shearing
torsion
bending
buckling


Transverse
Oblique
Butterfly
Spiral
Fracture Patterns
THE EXPERTS
Robson Forensic


Mechanical Loading of Bone
Compression
Tension
Shear
Torsion
Bending

Stress
(For Tension and Compression)

To compare specimens , the load is calculated per unit area.

Stress: $\sigma = F / A_o$

F: **is load**

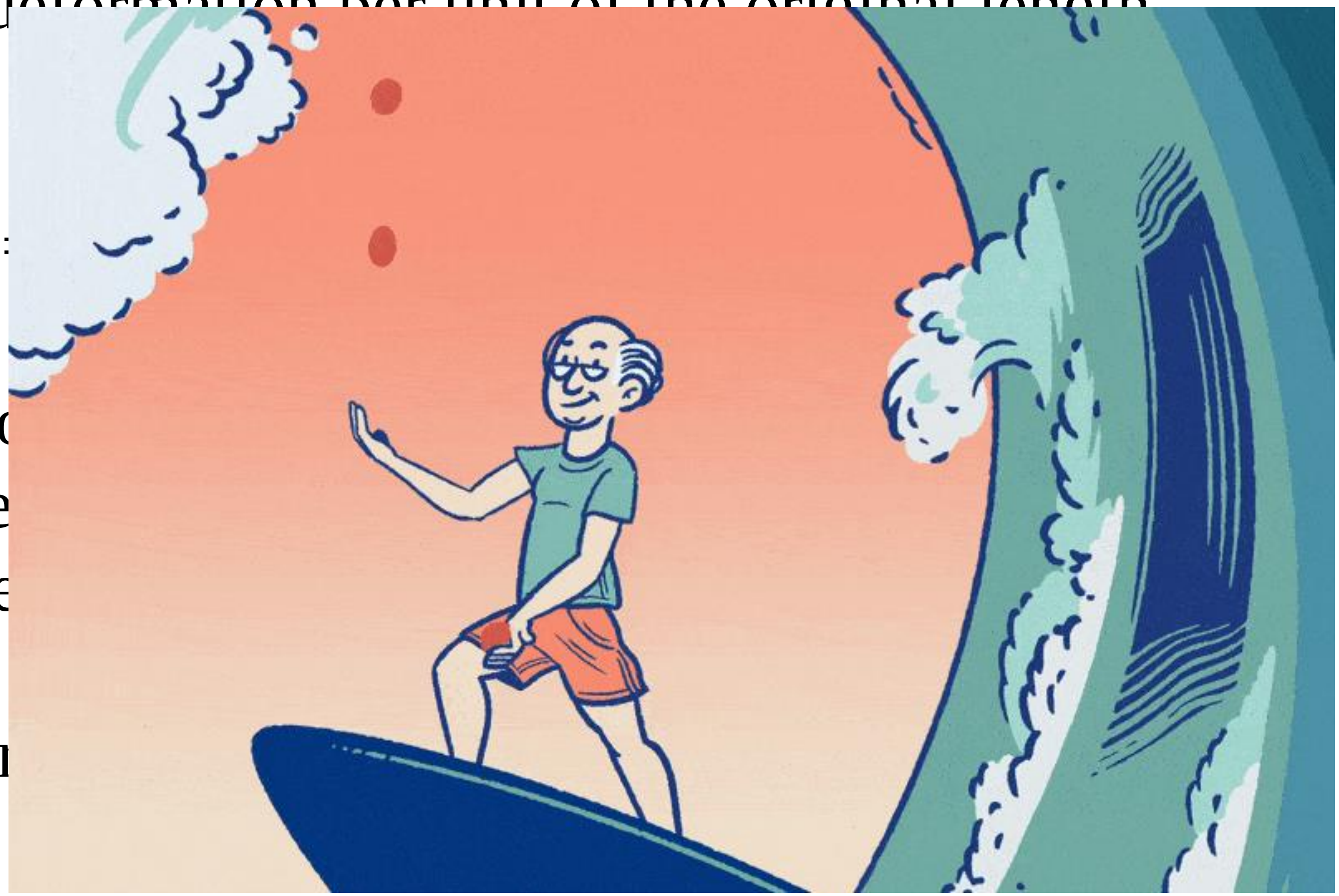
A_0: **cross-sectional area**

A_0 perpendicular to F before application of the load.

DIRECT STRAIN , ε

In each case, a force F produces a deformation x. In physics, the force is usually refer to stress and the deformation to strain. Strain is the deformation per unit of the original length.

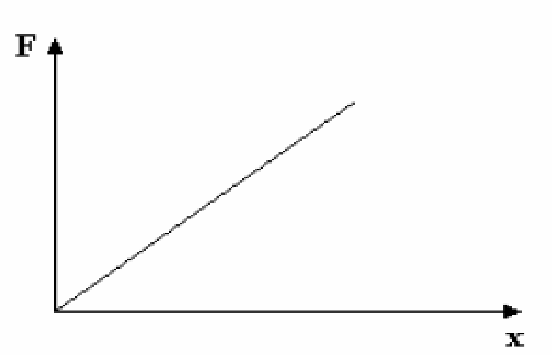
$Strain = \varepsilon =$



Strain has no
Most engine
they become
figures. It is
exponent for

MODULUS OF ELASTICITY (E)

•Elastic materials always spring back into shape when released. They also obey HOOK's LAW.



•This is the law of spring which states that deformation is directly proportional to the force. F/x = stiffness (N/m)

•The stiffness is different for the different material and different sizes of the material. We may eliminate the size by using stress and strain instead of force and deformation:

•If F and x is refer to the direct stress and strain , then

$$F = \sigma A \quad x = \varepsilon L \quad \text{hence} \quad \frac{F}{x} = \frac{\sigma A}{\varepsilon L} \quad \text{and} \quad \frac{FL}{Ax} = \frac{\sigma}{\varepsilon}$$

Stress
For Shear and Torsion

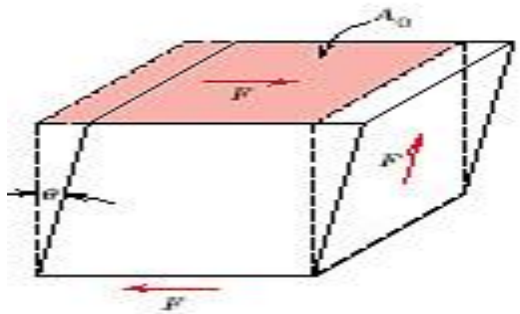
Shear stress: $\tau = F / A_o$

F is applied parallel to upper and lower faces each having area A_0.

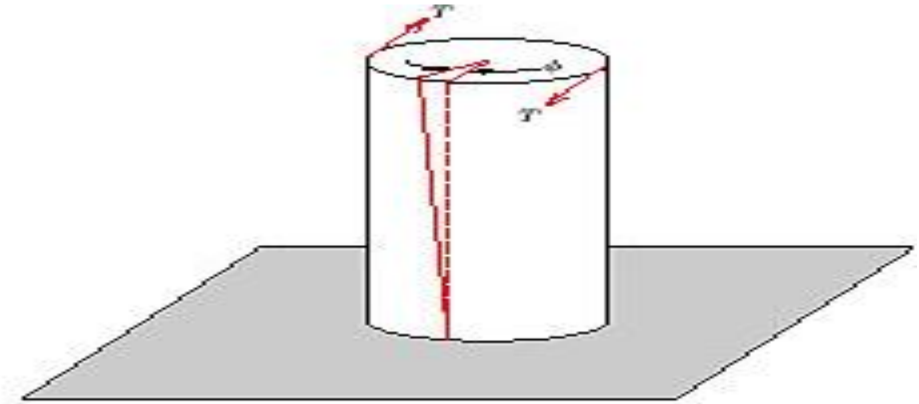
Shear strain: $\gamma = \tan\theta$ **(gamma)**

θ is strain angle

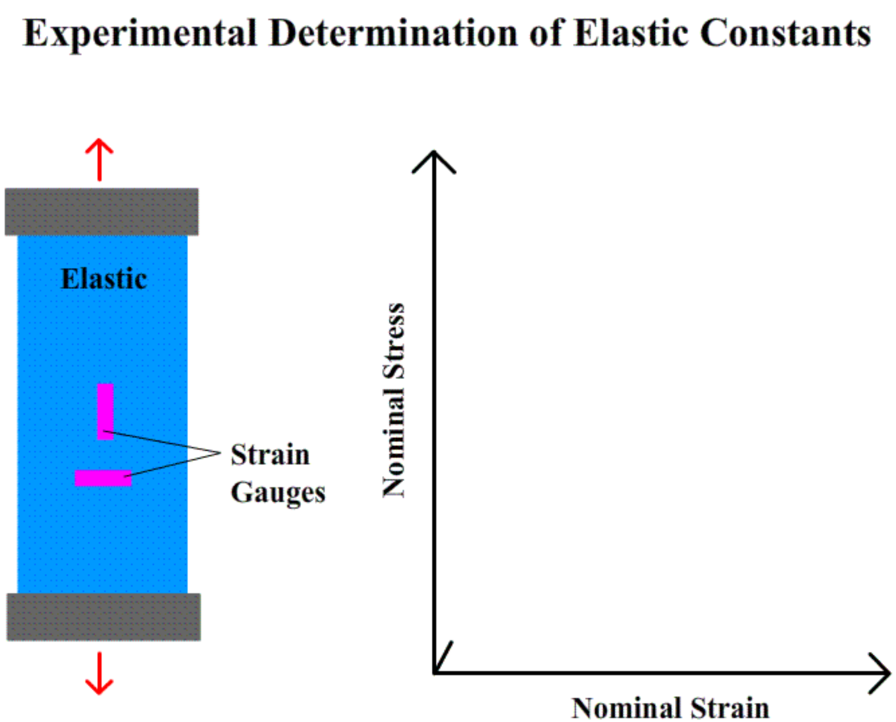
Shear



Torsion



Stress-Strain Behavior



Elastic deformation

Reversible:

(For small strains)

Stress removed → material returns to original size

Plastic deformation

Irreversible:

Stress removed → material does not return to original dimensions.

Modulus of Elasticity

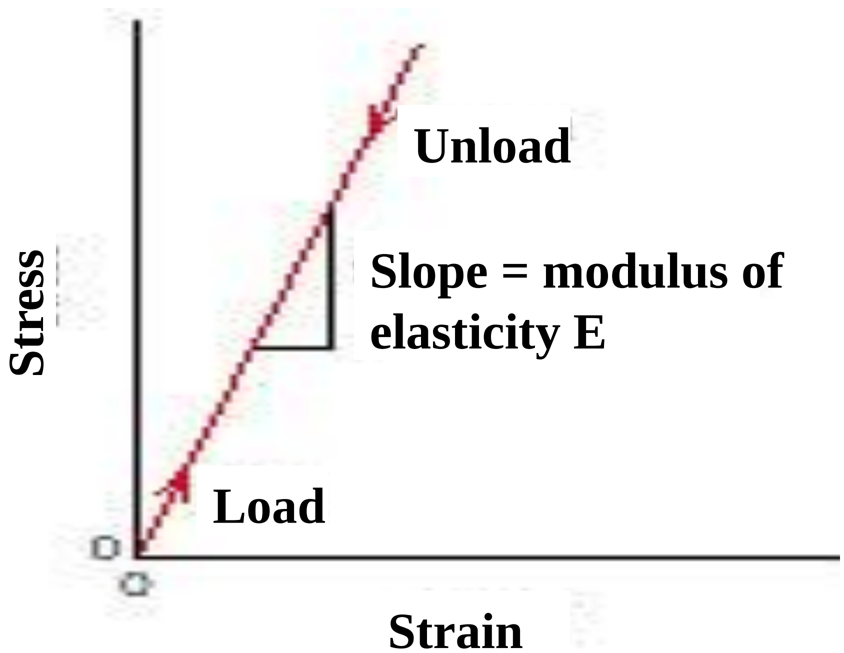
- The modulus of elasticity plays the role of the spring constant for solids.
- A material is elastic when it can take a large amount of strain before breaking.
- A brittle material breaks at a very low value of strain.

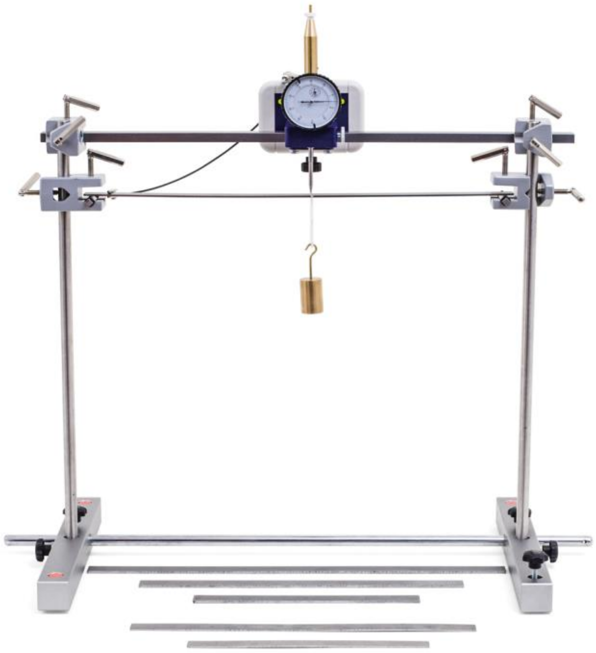
Elastic deformation

Gives Hook's law for Tensile Stress

$$\sigma = E\,\varepsilon$$

E = Young's modulus or modulus of elasticity (same units as σ, N/m^2 or Pa)

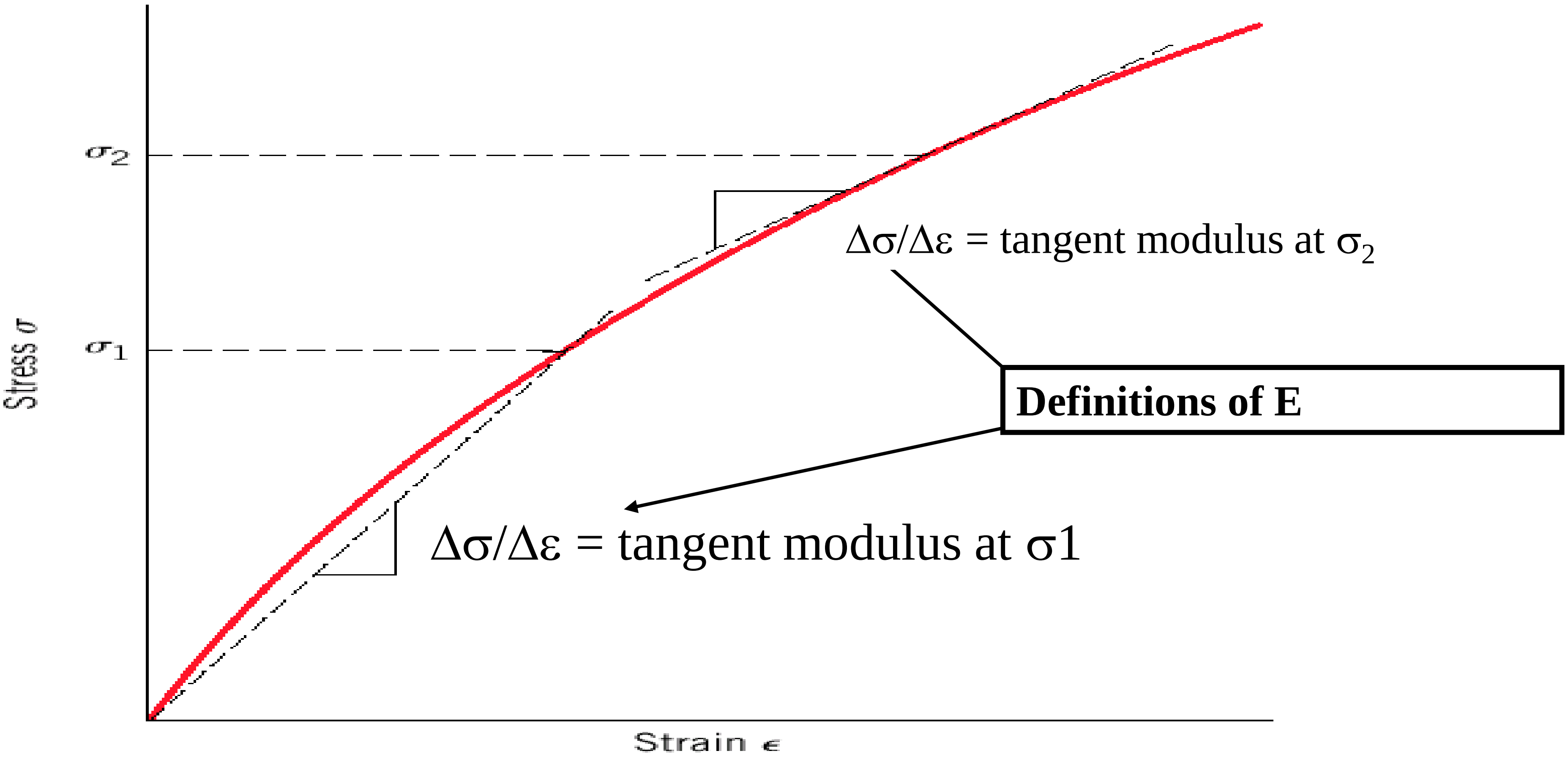




Higher E → higher "stiffness"

Nonlinear elastic behavior

In some materials (many polymers, concrete...), elastic deformation is not linear, but it is still reversible.



Stress and Strain

- **Stress: Intensity of the internal force, measured by force per unit area**
- **Strain: Elongation per unit length**
- **True Stress: Force divided by the actual cross sectional area**

Formulas for Stress and Strain

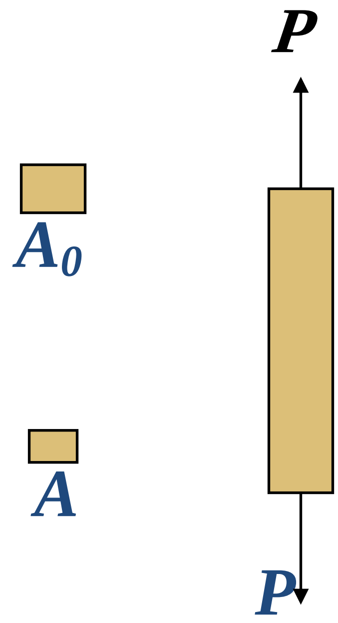
For an axially loaded member:

Nominal (or Engineering) Stress

$$\sigma = \frac{P}{A_o}$$

True Stress

$$\sigma = \frac{P}{A}$$



Where σ = stress, psi or pascal

P = magnitude of the applied force, N

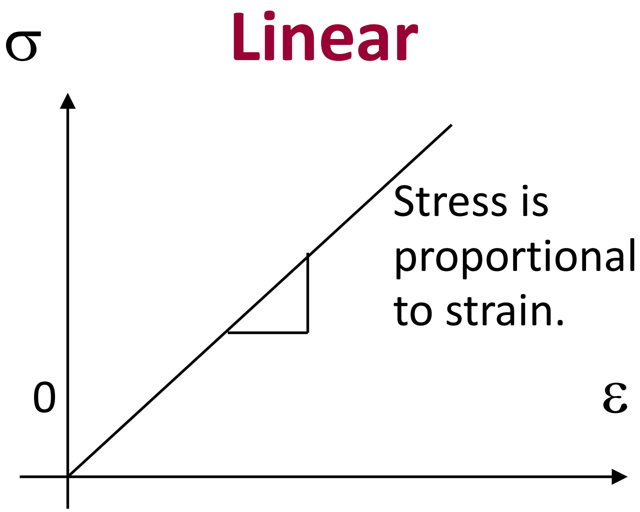
A_o = original cross sectional area, m^2

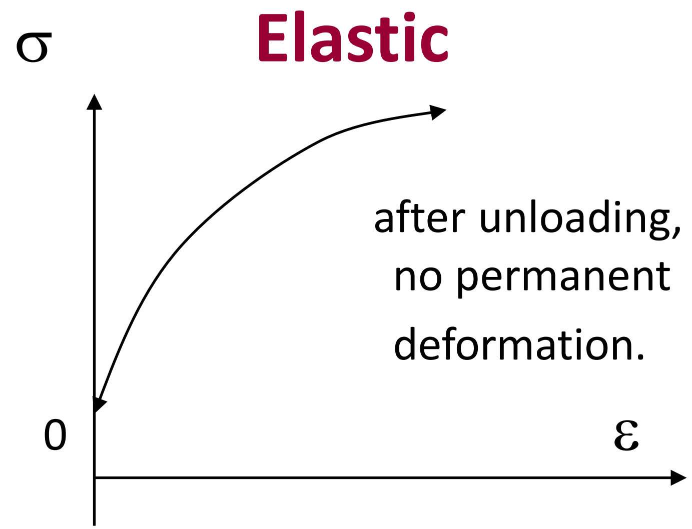
A = cross sectional area at the moment the stress is calculated, m^2

More Definitions

- **Stress vs. Strain Diagrams:** Plot of stress vs. strain for a given material.
- **Linear (Elastic) Range:** Range of stress-strain diagram in which stress is (generally) proportional to strain.
- **Nonlinear (Plastic) Range:** Range of stress-strain diagram in which stress is **NOT** proportional to strain.

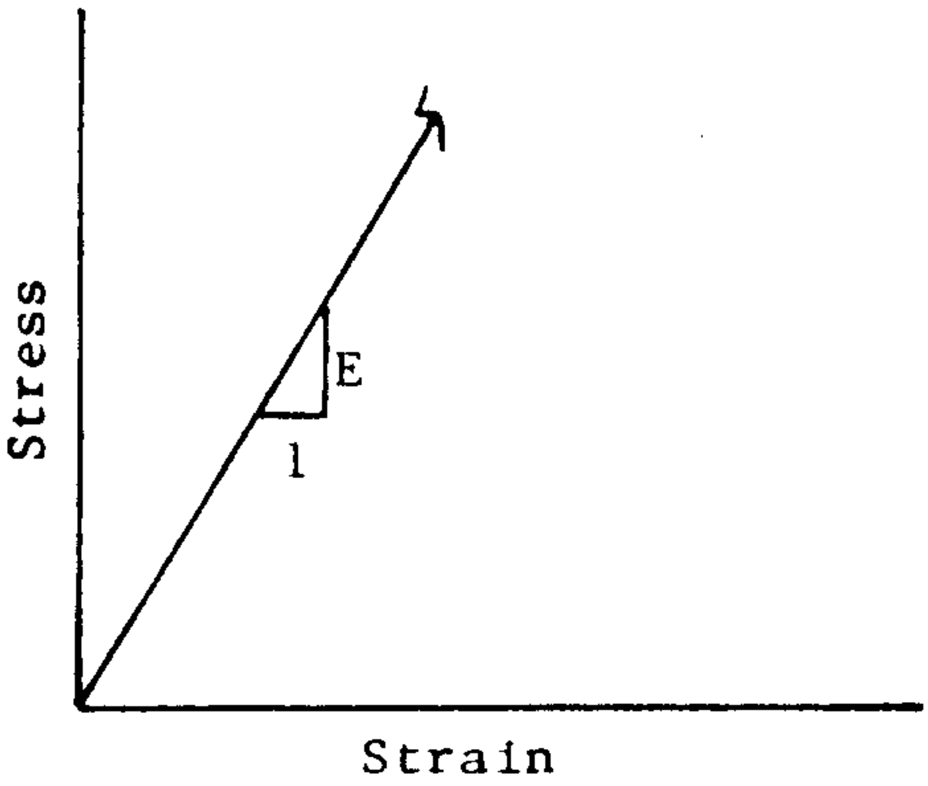
“Linear” and “Elastic”





The Linear (Elastic) Range

The **slope** of the line in the linear (elastic) range is the **elastic modulus, E**, and is the constant of proportionality between the stress and the strain.



Modulus of Elasticity

- Constant of proportionality (slope of a line) in elastic range.

$$E = \frac{\sigma}{\varepsilon}$$

Unit: N/m^2

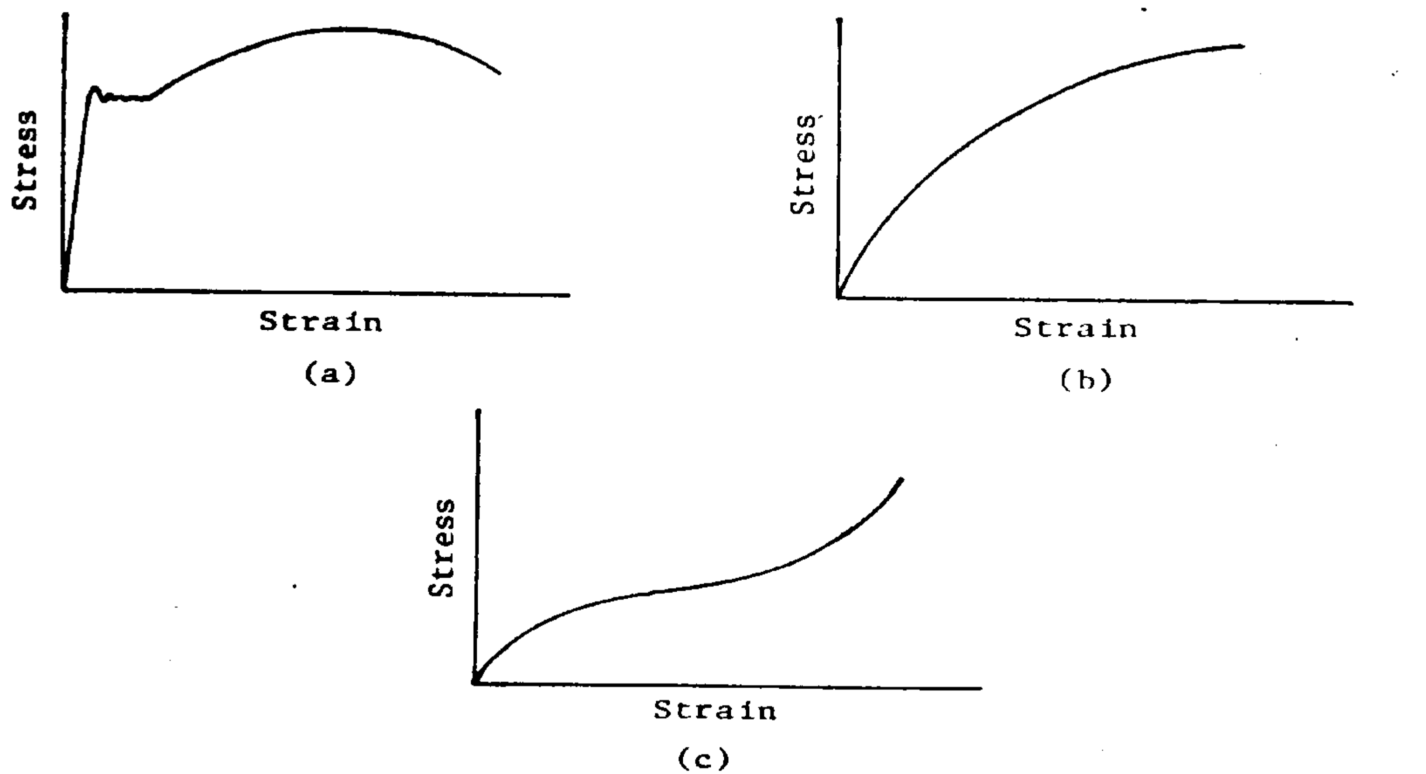
- It is also called as **Young's Modulus**.
- For a linear material, the relationship between stress and strain:

$$\sigma = E\varepsilon$$

and

$$\varepsilon = \frac{\sigma}{E}$$

Typical Stress-Strain Curves for Materials



(a) Mild Steel **(b) Iron**

(c) Rubber

Strength in Elastic Range

- **Proportional limit:** The point beyond which stress is no longer **proportional** to strain.
- **Elastic Limit:** The point beyond which **permanent deformation** will result when the load is removed.

To use our formula we need to define what we mean by Stress and Strain.

STRESS is the same idea as PRESSURE. In fact it is the same formula:

$$\text{Stress} = \frac{\text{Force}}{\text{Area}}$$

STRAIN is a measure of how much the object deforms. We divide the change in the length by the original length to get strain:

$$\text{Strain} = \frac{\Delta L}{L_0}$$

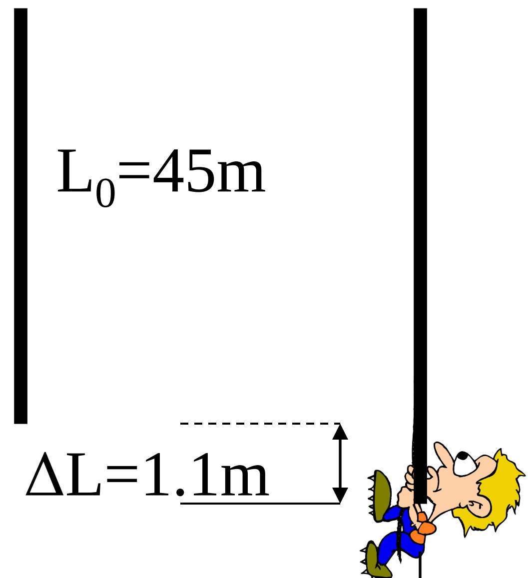
Now we can put these together to get our formula for the Young's Modulus:

$$Y = \frac{F/A}{\Delta L/L_0}$$

Problem : A nylon rope used by mountaineers elongates 1.1 m under the weight of a 65 kg climber. If the rope is initially 45 m in length and 7 mm in diameter, what is Young's modulus for this nylon?

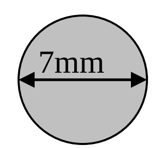
A couple of quick calculations and we can just plug in to our formula:

$$Y = \frac{F/A}{\Delta L / L_0}$$



$$Y = \frac{637N / 3.85 \cdot 10^{-5} m^2}{1.1m / 45m} = \frac{1.65 \cdot 10^7 \frac{N}{m^2}}{0.024} = \boxed{6.88 \cdot 10^8 \frac{N}{m^2}}$$

$$F = mg = (65kg)\left(9.8 \frac{m}{s^2}\right) = 637N$$

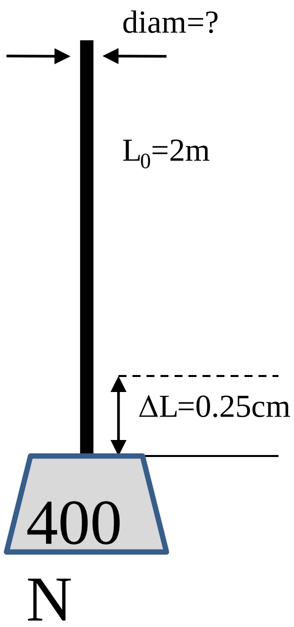


$$A = \pi r^2 = \pi (3.5 \cdot 10^{-3} m)^2 = 3.85 \cdot 10^{-5} m^2$$

Don't forget to cut the diameter in half.

Problem : A steel wire 2 m long with circular cross-section must stretch no more than 0.25cm when a 400 N weight is hung from one of its ends. What minimum diameter must this wire have?

We have most of the information for our formula. We can look up Young's modulus for steel in a table: $Y_{steel} = 2 \cdot 10^{11} \frac{N}{m^2}$



$$Y = \frac{F/A}{\Delta L / L_0}$$

The only piece missing is the area – we can rearrange the formula

$$A = \frac{F \cdot L_0}{Y \cdot \Delta L}$$

$$A = \frac{(400N)(2m)}{\left(2 \cdot 10^{11} \frac{N}{m^2}\right)(0.0025m)} = 1.6 \cdot 10^{-6} m^2$$

One last step – we need the diameter, and we have the area:

$$\pi r^2 = A_{circle} \Rightarrow r = \sqrt{\frac{1.6 \cdot 10^{-6} m^2}{\pi}} = 7.14 \cdot 10^{-4} m$$

double the radius to get the diameter:

$$d = 1.4 \cdot 10^{-3} m = 1.4mm$$