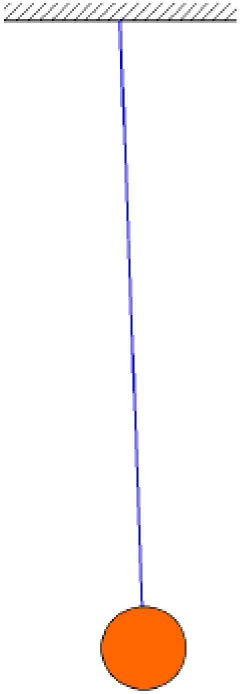
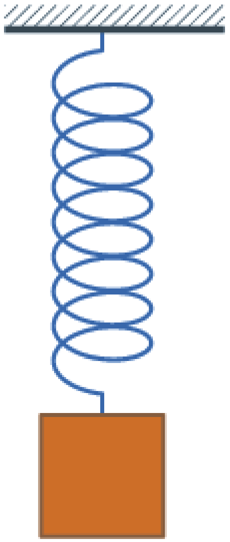
Oscillations and Waves



Kinematics of simple harmonic motion (SHM)



Periodic Motion

- Objects that move back and forth periodically are described as ***oscillating***.
- These objects move past an ***equilibrium position***, *O* (where the body would rest if a force were not applied) and their ***displacement*** from this position changes with time.
- If the time period is independent of the maximum displacement, the motion is ***isochronous***.

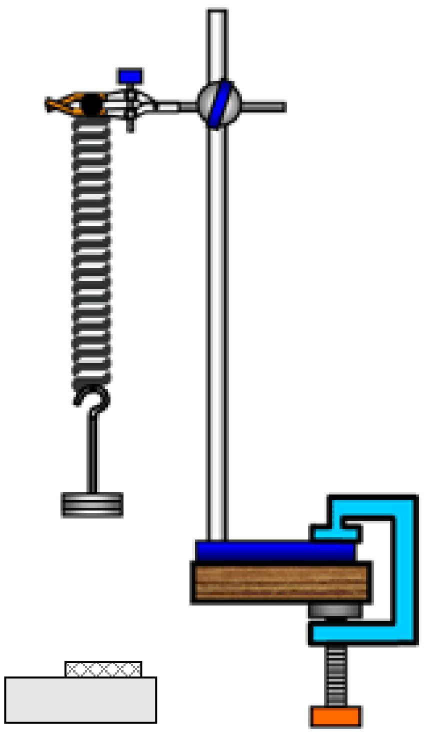
E.g.

-Oscillating pendulums, watch springs or atoms can all be used to measure time

Properties of oscillating bodies

A *time trace* is a graph showing the variation of displacement against time for an oscillating body.

Demo: Producing a time-trace of a mass on a spring.



Distance sensor connected to computer

Amplitude (x_0): The maximum displacement (in m) from the equilibrium position (Note that this can reduce over time due to damping).

Cycle: One complete oscillation of the body.

Period (T): The time (in s) for one complete cycle.

Frequency (f): The number of complete cycles made per second (in Hertz or s^{-1}). (*Note: $f = 1 / T$*)

Angular frequency (ω): Also called angular speed, in circular motion this is a measure of the rate of rotation. In periodic motion it is a constant (with units s^{-1} or rad s^{-1}) given by the formula…

$$\boxed{\omega = \frac{2\pi}{T} = 2\pi f}$$

Q.

Calculate the angular speed of the hour hand of an analogue watch (in radians per second).

Angle in one hour = 2π radians

Time for one revolution = 60 x 60 x 12 = 43200s

$$\omega = \frac{2\pi}{T} = \frac{1.45 \times 10^{-3}\ \text{rad s}^{-1}}{43200\text{s}}$$

1 rad = 180°/π = 57.295779513°

Determining the Frequency of Medical Ultrasound

Ultrasound machines are used by medical professionals to make images for examining internal organs of the body. An ultrasound machine emits high-frequency sound waves, which reflect off the organs, and a computer receives the waves, using them to create a picture. We can use the formulas presented in this module to determine the frequency, based on what we know about oscillations. Consider a medical imaging device that produces ultrasound by oscillating with a period of $0.400 \mu s$. What is the frequency of this oscillation?

The period (*T*) is given and we are asked to find frequency (*f*).

Solution

Substitute $0.400 \mu s$0.400μs for *T* in $f=\frac{1}{T}$f=1T:

$f=\frac{1}{T}=\frac{1}{0.400\times10^{-6}s}$.f=1T=10.400×10−6s.

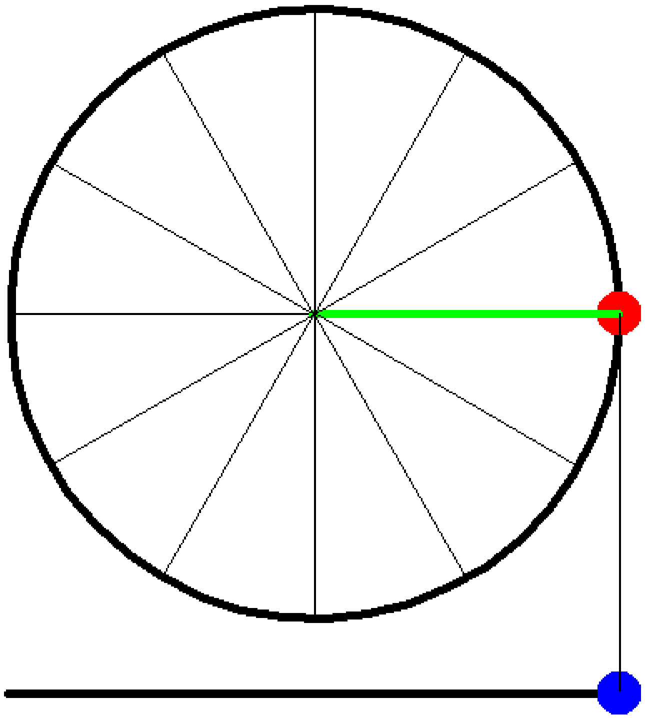
Solve to find

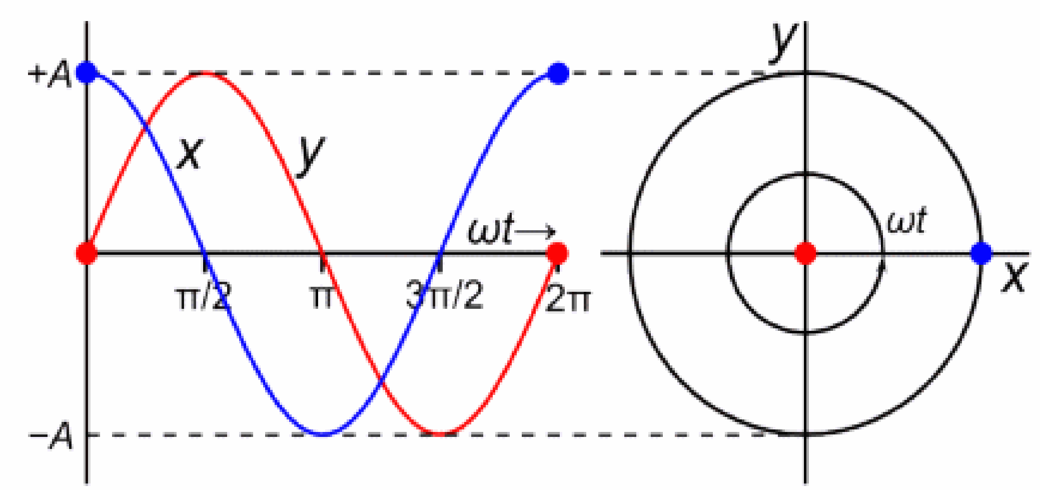
$f=2.50\times10^{6}Hz$.f=2.50×10⁶Hz.

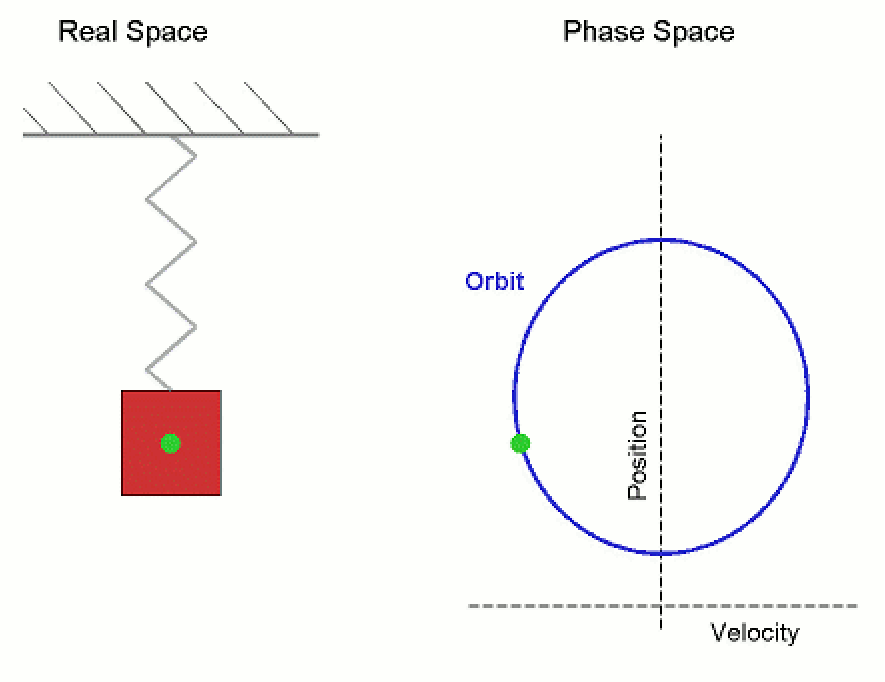
This frequency of sound is much higher than the highest frequency that humans can hear (the range of human hearing is 20 Hz to 20,000 Hz); therefore, it is called ultrasound. Appropriate oscillations at this frequency generate ultrasound used for non-invasive medical diagnoses, such as observations of a fetus in the womb.

Simple Harmonic Motion (SHM)

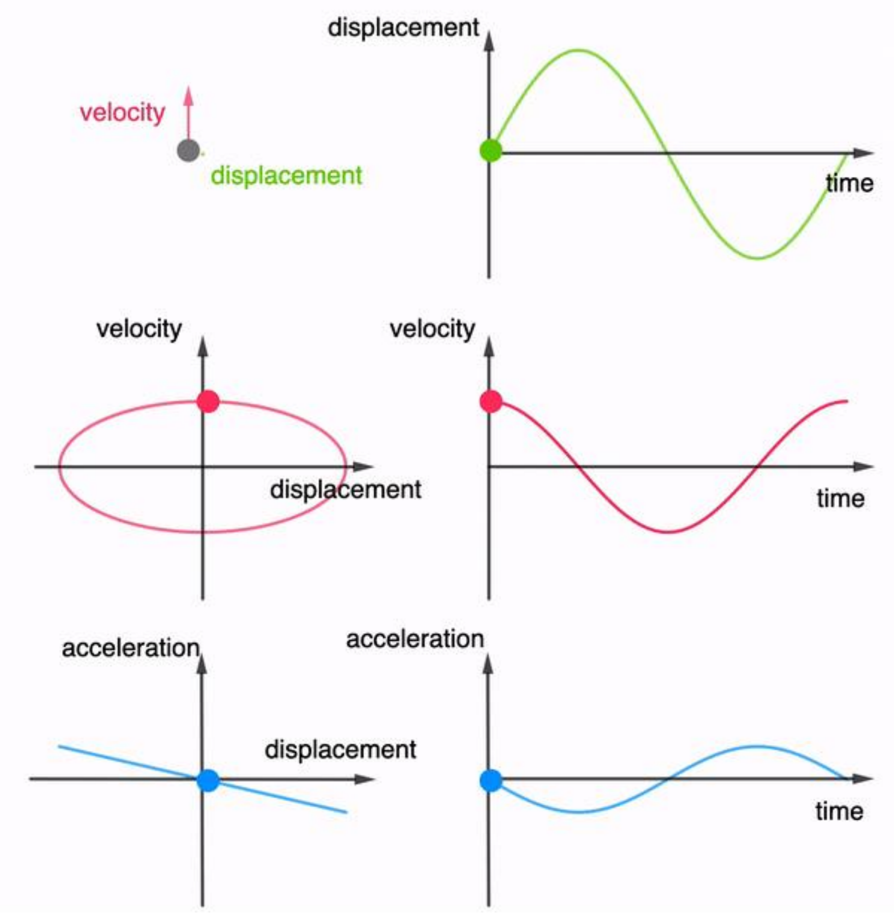
Consider this example



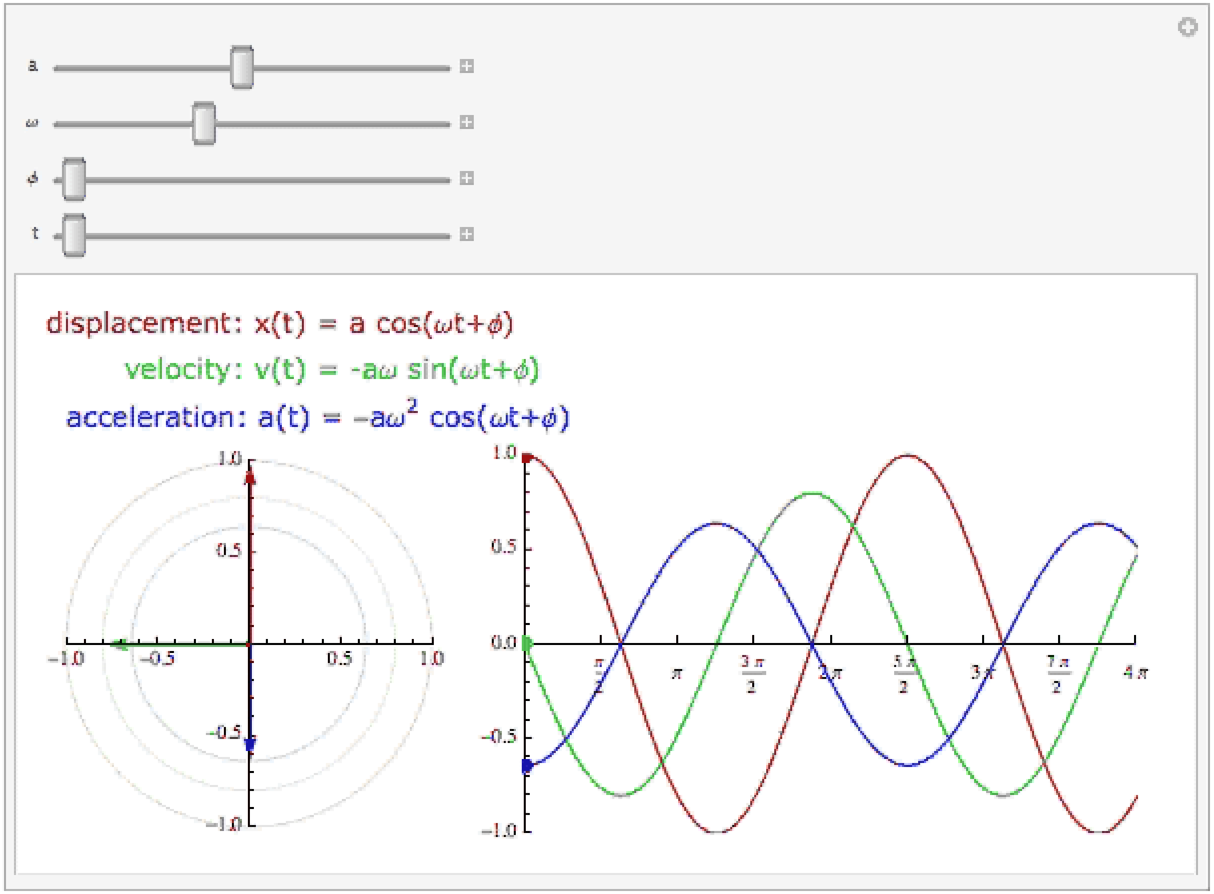



Real Space
Phase Space
Orbit
Position
Velocity

SHM- Velocity and Acceleration



SHM- Velocity and Acceleration



- At time, $t = 0$ the object is at point M (Figure 0) and after time t it moves to point N , therefore the expression for displacement, x_1 is given by

$$x_1 = A\sin\theta_1 \text{ where } \theta_1 = \theta + \phi \text{ and } \theta = \omega t$$
$$x_1 = A\sin(\omega t + \phi)$$

- In general the **equation of displacement** as a function of time in SHM is given by

displacement from equilibrium position

$$\boxed{x = A\sin\overbrace{(\omega t + \phi)}^{\textbf{phase}}} \quad \text{..................} \quad (4)$$

amplitude

angular frequency

time

Initial phase angle (phase constant)

- The S.I. unit of displacement is **meter (m)**.

Phase

- It is the time-varying quantity $(\omega t + \phi)$.
- Its unit is **radian**.

Conclusion:

From these graphs we can see…

- Whenever x is positive, a is negative.
- a is proportional to x (as they both have maximum values at the same times).

Thus we can say…

$$a \propto -x$$

$$\boxed{a = -\omega^2 x}$$

… where ω is a constant called the angular frequency (s^{-1}).

This is the defining equation for SHM

Conditions for SHM

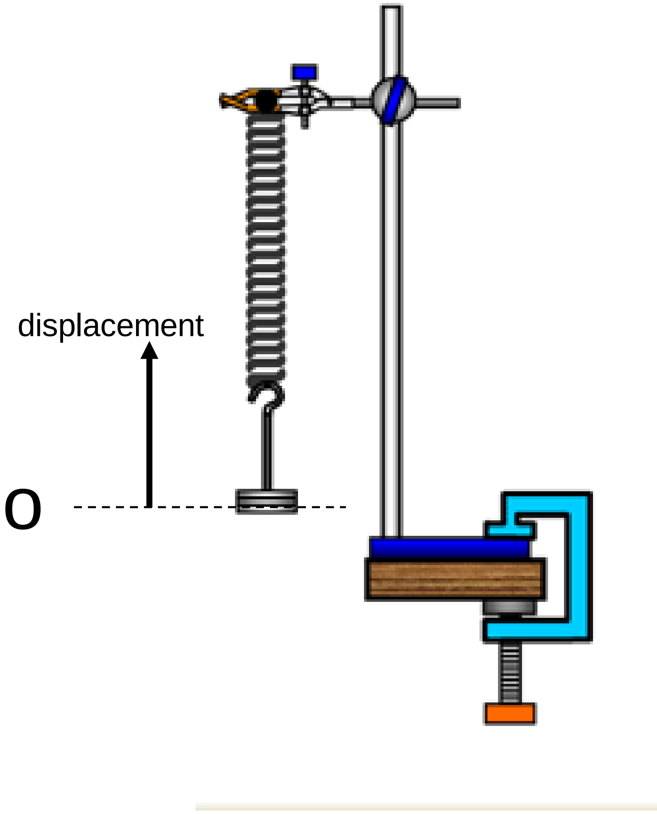
From the equation $a = -\omega^2 x$ we can say Simple harmonic motion is taking place if…

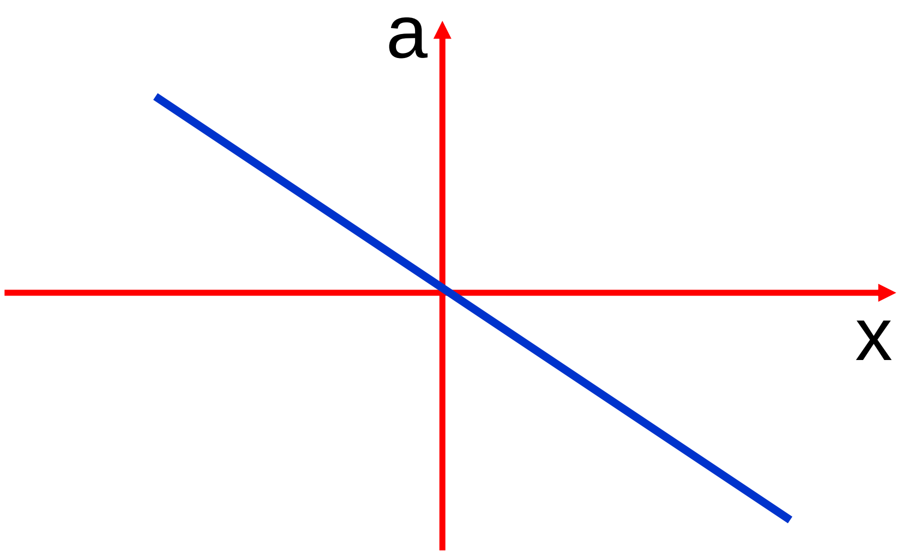
i. acceleration is always proportional to the displacement from the equilibrium point.

ii. acceleration is always directed towards the equilibrium position (i.e. opposite direction to the displacement).

Q1

Sketch a graph of *acceleration* against *displacement* for the oscillating mass shown (take upwards as positive.





Further equations for SHM

If $a = -\omega^2 x$ then $\dfrac{d^2x}{dt^2} = -\omega^2 x$

There are many sets of possible mathematical solutions to this differential equation. Here are two…

$$x = A \sin(\omega t)$$
$$v = \omega A \cos(\omega t)$$
$$a = -\omega^2 A \sin(\omega t)$$

$$x = A \cos(\omega t)$$
$$v = -\omega A \sin(\omega t)$$
$$a = -\omega^2 A \cos(\omega t)$$

If velocity needs to be calculated in terms of displacement only, we can also use…

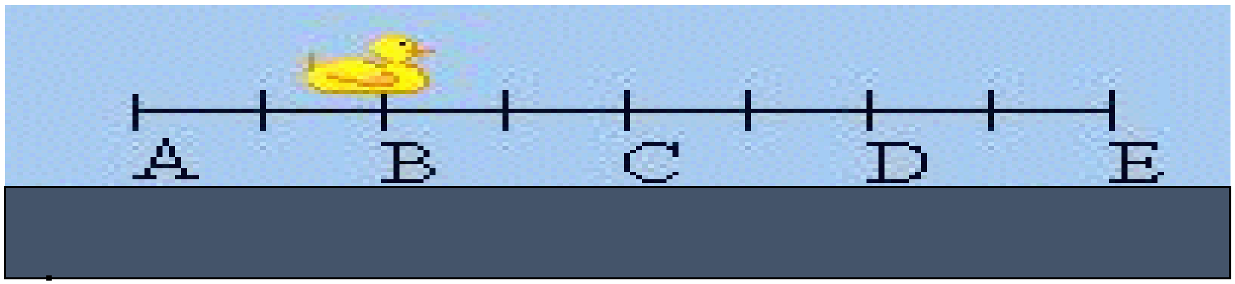
$$v = \omega \sqrt{(A^2 - x^2)}$$

So what is the velocity at maximum and zero displacements?

Does this agree with your understanding of shm?

Q2

Consider this duck, oscillating with SHM…



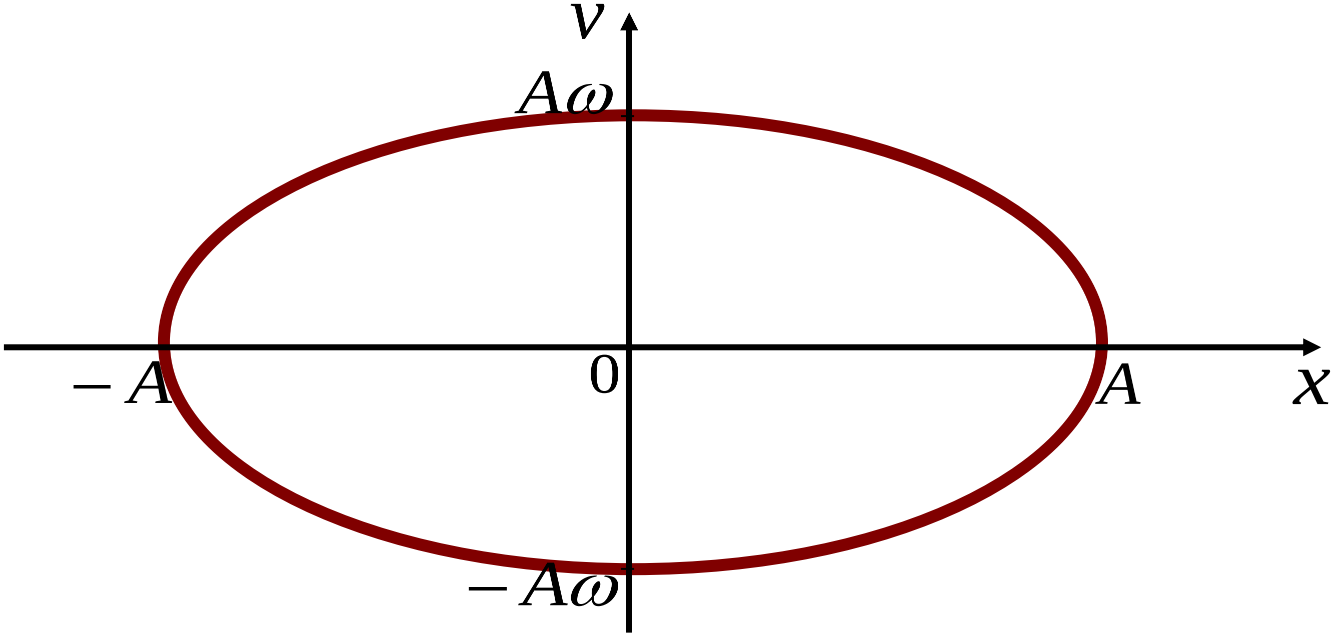
Where is…

i.	Displacement at a maximum?	A and E
ii.	Displacement zero?	C
iii.	Velocity at a maximum?	C
iv.	Velocity zero?	A and E
v.	Acceleration at a maximum?	A and E
vi.	Acceleration zero?	C

- From the relationship between velocity and displacement,

$$v = \omega\sqrt{A^2 - x^2}$$

thus the graph of **velocity against displacement (*v*-*x*)** is shown in Figure 9.9.

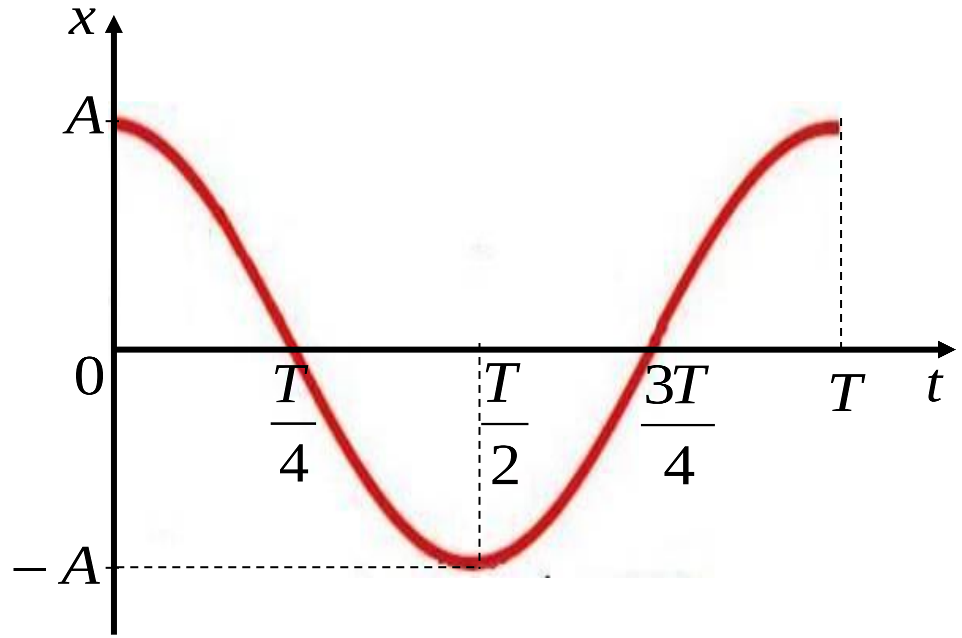


- For examples:

  a. At $t = 0$ s, $x = +A$

  Equation: $x = A\sin\left(\omega t + \frac{\pi}{2}\right)$ OR $x = A\cos(\omega t)$

  Graph of x against t:

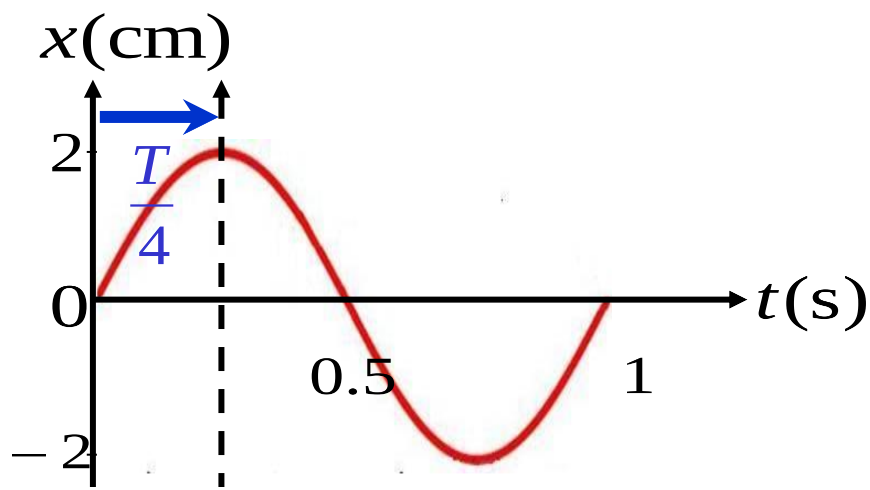


How to sketch the x against t graph when $\phi \neq 0$

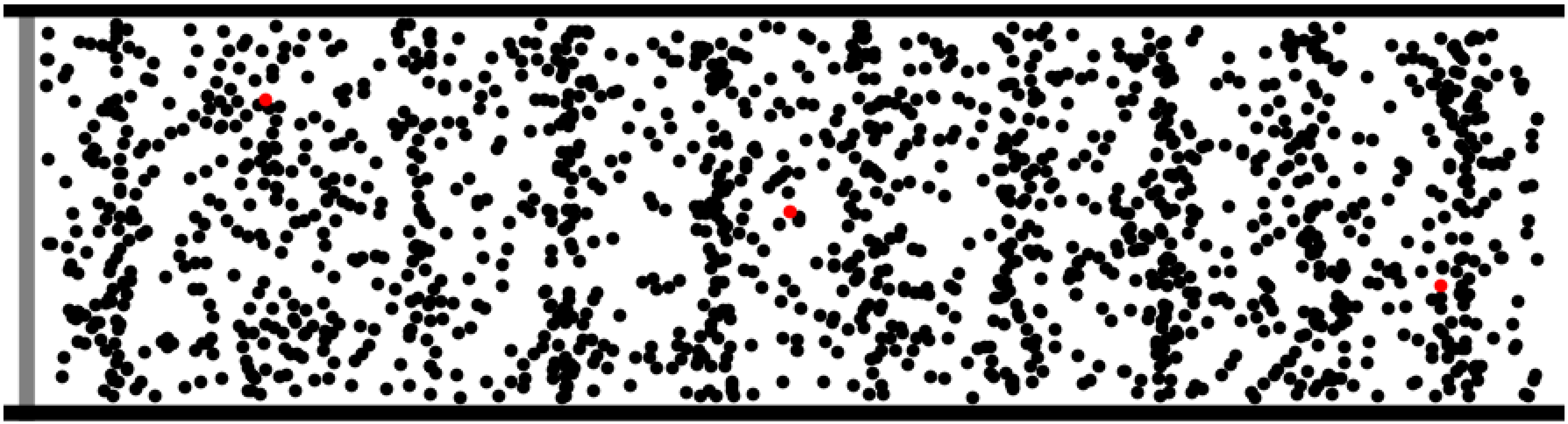
Sketch the x against t graph for the following expression:

$$x = 2\,\mathrm{cm}\sin\left(2\pi t + \frac{\pi}{2}\right)$$

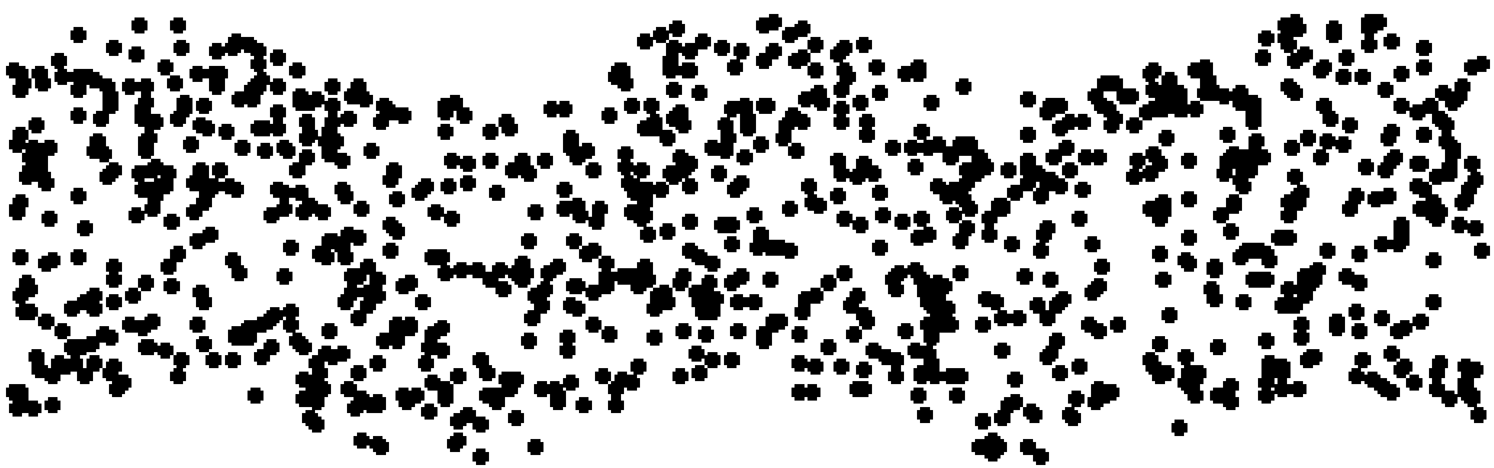
- From the expression,
  - the amplitude, $A = 2\ \mathrm{cm}$
  - the angular frequency, $\omega = 2\pi\ \mathrm{rad\,s^{-1}} = \frac{2\pi}{T} \Rightarrow T = 1\ \mathrm{s}$
- Sketch the x against t graph for equation $x = 2\sin(2\pi t)$



Longitudinal Waves - Kundt's Tube



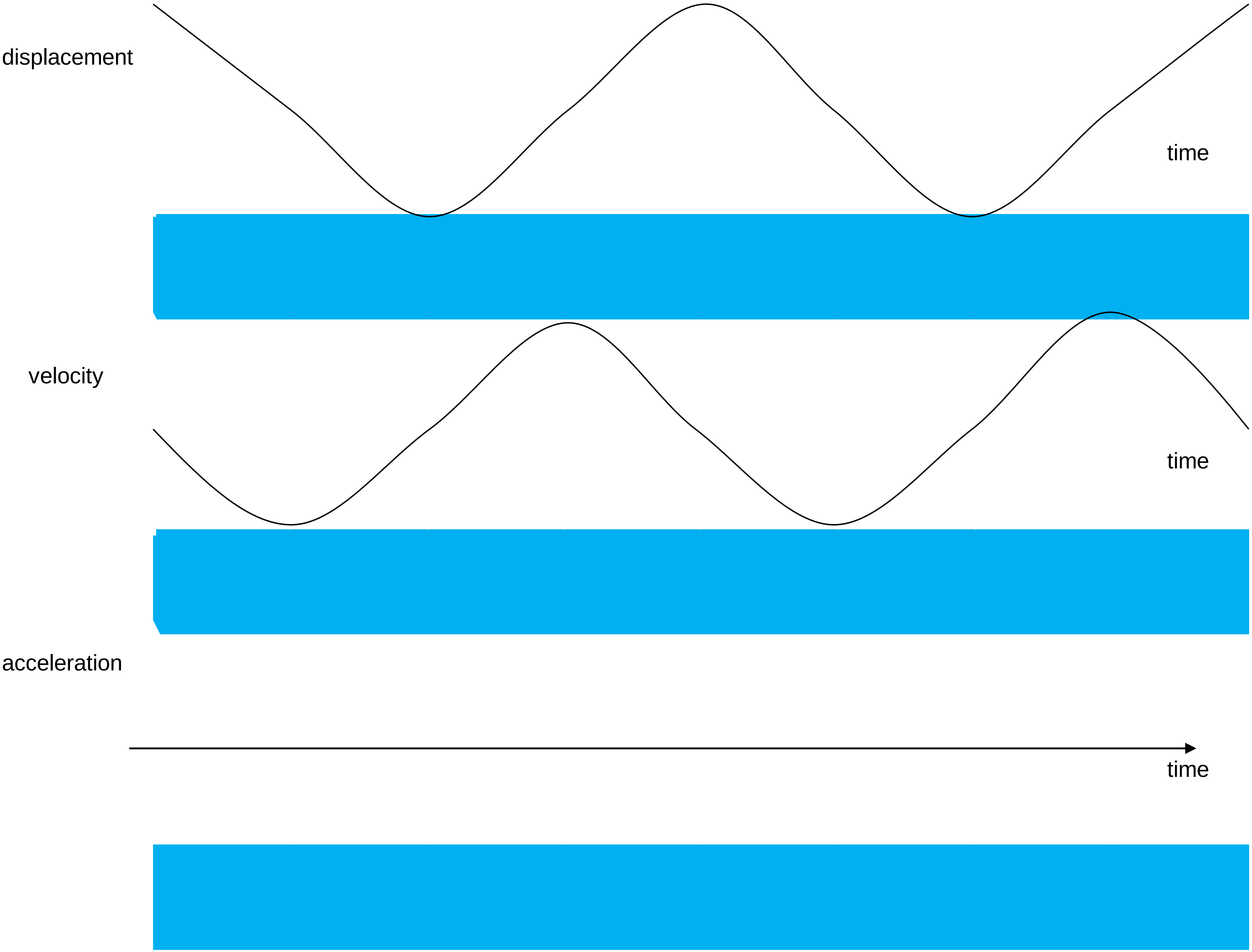
transverse waves



Q.

Sketch graphs that would be represented by the two sets of SHM equations

$x = x_0 \sin(\omega t)$	$x = x_0 \cos(\omega t)$
$v = \omega x_0 \cos(\omega t)$	$v = -\omega x_0 \sin(\omega t)$
$a = -\omega^2 x_0 \sin(\omega t)$	$a = -\omega^2 x_0 \cos(\omega t)$


displacement
time
velocity
time
acceleration
time