# Collisions in One Dimension

## The aim:--------------------------------------------------------------------------------

## Theory

### Conservation of Momentum

Let the mass of a particle be $m$ and its velocity $\mathbf{v}$. The momentum $\mathbf{p}$ is defined as $\mathbf{p} = m\mathbf{v}$ and Newton's 2nd law is $\mathbf{F} = \frac{d\mathbf{p}}{dt}$, where $\mathbf{F}$ is the net force on $m$. If $\mathbf{F}= 0$, then $\mathbf{p}$ = constant (1st law).

For a system of particles, we identify each particle with a subscript $i$ so that $\mathbf{F}_i = d\mathbf{p}_i/dt$. If both sides of this equation are summed over all the particles, we get:

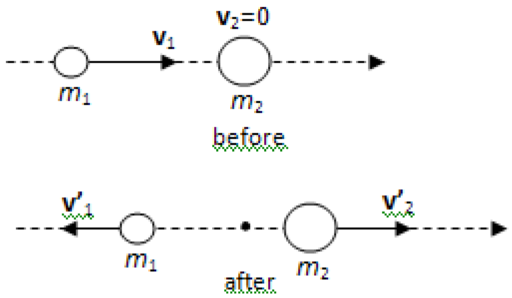
$$\sum_{i=1}^{n}\mathbf{F}_i = \mathbf{F} = \frac{d}{dt}\sum_{i=1}^{n}\mathbf{p}_i = \frac{d}{dt}\mathbf{P}$$

where $n$ is the number of particles and $\mathbf{P}$ is the total momentum of the system. In the sum over $\mathbf{F}_i$, the internal forces (that is, forces between the particles) can be excluded as Newton's 3rd law guarantees that they cancel out.

A collision is said to be elastic if the total KE of the two objects is the same after the collision as before the collision. If the KE is less after the collision the collision is called inelastic. If the two objects stick together the collision is called completely or perfectly inelastic.

### Two Particle One-dimensional Elastic Collisions

Consider a system of two particles where the first, of mass $m_1$ and initial velocity $\mathbf{v}_1$, collides with the second particle of mass $m_2$ which is initially **at rest**. Denote the velocities after the collision with primes, see figure. As we are working only in one dimension, we will deal only with velocities in that dimension and vector notation is not needed: **but**, velocities in the positive sense are positive and those in opposite direction are negative.




For an elastic collision, conservation of energy and momentum give, respectively:

$$\frac{1}{2}m_1\upsilon_1^2 = \frac{1}{2}m_1\upsilon_1'^2 + \frac{1}{2}m_2\upsilon_2'^2 \qquad \text{and} \qquad m_1\upsilon_1 = m_1\upsilon_1' + m_2\upsilon_2'$$

These equations can be solved for $\upsilon_1'$ and $\upsilon_2'$ to give:

$$\upsilon_1' = \frac{m_1 - m_2}{m_1 + m_2}\upsilon_1 \qquad \text{and} \qquad \upsilon_2' = \frac{2m_1}{m_1 + m_2}\upsilon_1$$

## Procedure

Run the simulation file **(collision-lab_en)**.

**(A) Special cases**

1- Consider the following special cases:

- **a-** The two masses are equal, $m_1 = m_2$, traveling toward each other with different velocities.
- **b-** The two masses are equal, $m_1 = m_2$ and the second particle is initially at rest $\upsilon_2' = 0$.
- **c-** The second object is much more massive than the first $m_2 >> m_1$ and the second particle is initially at rest $\upsilon_2 = 0$.
- **d-** The second object is much lighter than the first $m_2 << m_1$ and the second particle is initially at rest $\upsilon_2 = 0$.

For each case:

2- Set the velocities before the collision $\upsilon_1$ (also $\upsilon_2$ in case **a**).

3- Click play and after collision, record $\upsilon_1'$ and $\upsilon_2'$.

4- Calculate the momentum and the kinetic energy of the system before and after collision.

5- Tabulate the results as in Table (A).

<u>NOTE:</u> All (–)ve signs should be considered for the velocities and momenta.

**(B) Determination of mass ratio**

1- Set $m_1$ = 1 kg and $m_2$ = 2 kg. With initial velocities give in Table (B), get the final velocities $\upsilon_1'$ and $\upsilon_2'$ and calculate $\Delta\upsilon_1$ and $\Delta\upsilon_2$.
2- Plot a graph of $\Delta\upsilon_2$ against $\Delta\upsilon_1$.
3- From the graph, calculate the ratio between the two masses and its percentage error.

**Table (A)**

| Special Cases | Before Collision | | | | | | After Collision | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $m_1$(kg) | $m_2$(kg) | $\upsilon_1$(m/s) | $\upsilon_2$ (m/s) | **P**(kgm/s) | $KE_{sys}$(J) | $\upsilon_1'$ (m/s) | $\upsilon_2'$ (m/s) | **P**(kgm/s) | $KE_{sys}$(J) |
| **a** | **1.5** | **1.5** | | | | | | | | |
| **b** | **1.5** | **1.5** | | **0** | | | | | | |
| **c** | **0.1** | **3** | | **0** | | | | | | |
| **d** | **3** | **0.1** | | **0** | | | | | | |

**Table (B)**

***m*₁ = 1 kg ,*m*₂ = 2 kg**

| $\upsilon_1$(m/s) | $\upsilon_2$ (m/s) | $\upsilon_1'$(m/s) | $\upsilon_2'$ (m/s) | $\Delta\upsilon_1$(m/s) | $\Delta\upsilon_2$(m/s) |
|---|---|---|---|---|---|
| **0.5** | **0** | | | | |
| **1** | **0** | | | | |
| **1.5** | **0** | | | | |
| **2** | **0** | | | | |
| **2.5** | **0** | | | | |
| **3** | **0** | | | | |

## Self-Evaluation

1. **Why all (–)ve signs should be considered for the velocities and momenta? Ans.…………**
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2. **Is the mass ratio that you have calculated gravitational or inertial? Ans. …………………**
**………………………………..…………………………………………………………………...**

**3- A collision is elastic if:**
(a) The final velocities are zero. (b) The final momentum is zero. (c) The objects stick together.
(d) The total kinetic energy is conserved. (e) The final kinetic energy is zero.

**4- Momentum will be conserved in a two-body collision only if**

(a) Both bodies come to rest. (b) The collision is perfectly elastic. (c) The kinetic energy of the system is conserved. (d) The net external force acting on the two-body system is zero. (e) The internal forces of the two body system cancel in action-reaction pairs.

**5- Two objects of equal mass traveling toward each other with equal speeds undergo a head on collision. Which one of the following statements concerning their velocities *after the collision* is *necessarily* true?**
(a) They will exchange velocities. (b) Their velocities will be zero. (c) Their velocities will be reduced.
(d) Their velocities may be zero. (e) Their velocities will be unchanged.