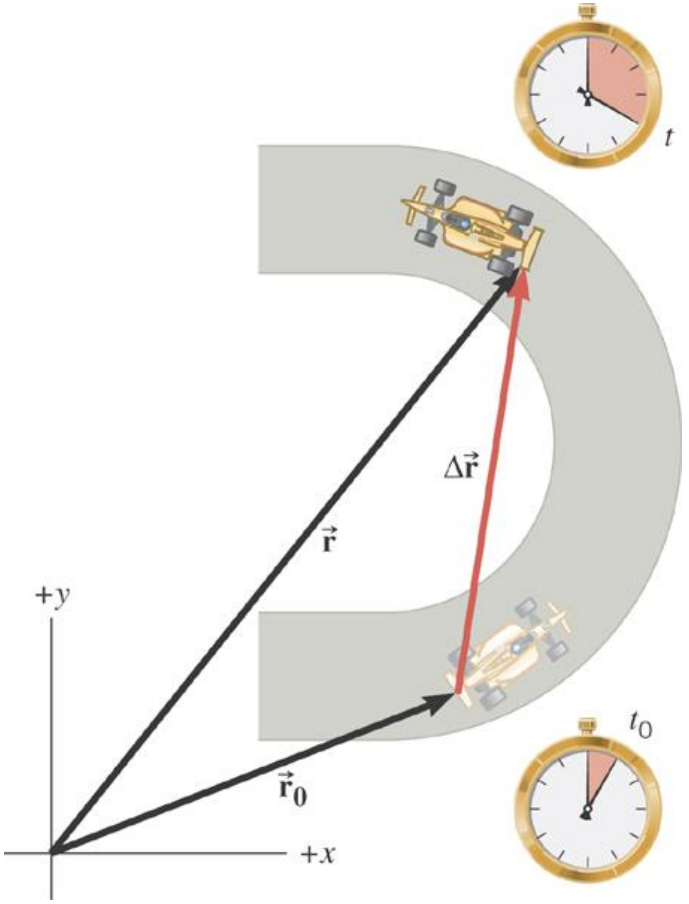
Kinematics of Particle in Two Dimensions

2.1 Displacement, Velocity, and Acceleration

$$\vec{\mathbf{r}}_o = \text{initial position}$$

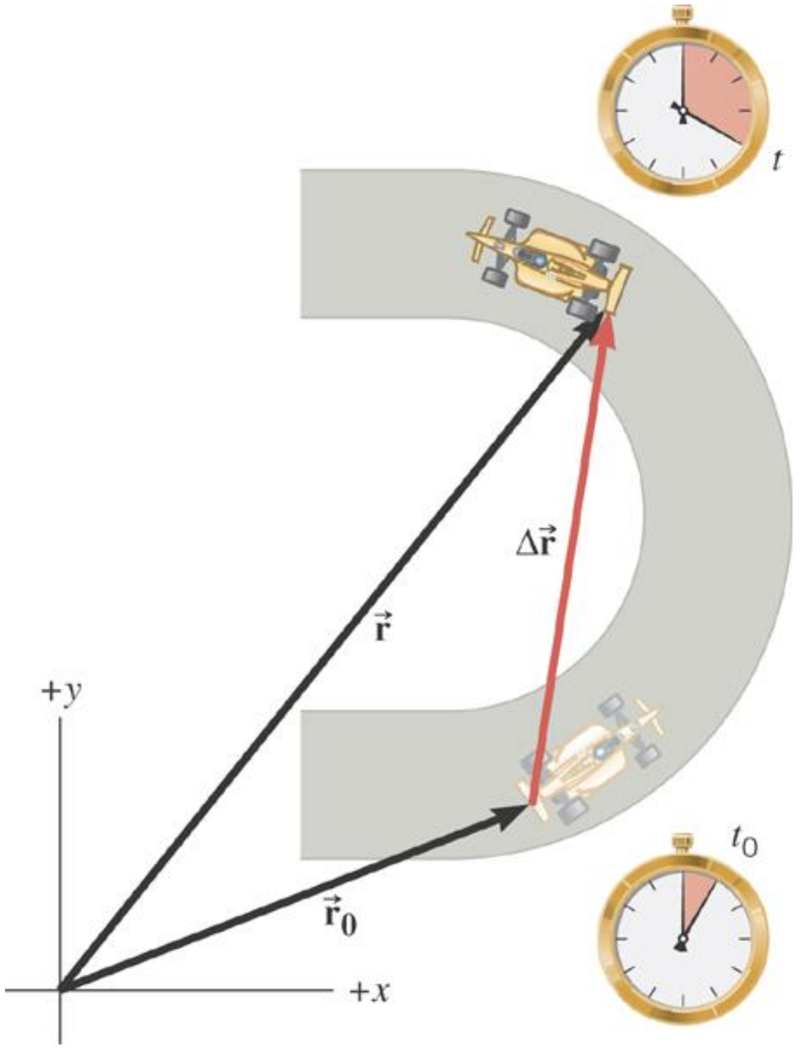
$$\vec{\mathbf{r}} = \text{final position}$$

$$\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}} - \vec{\mathbf{r}}_o = \text{displacement}$$



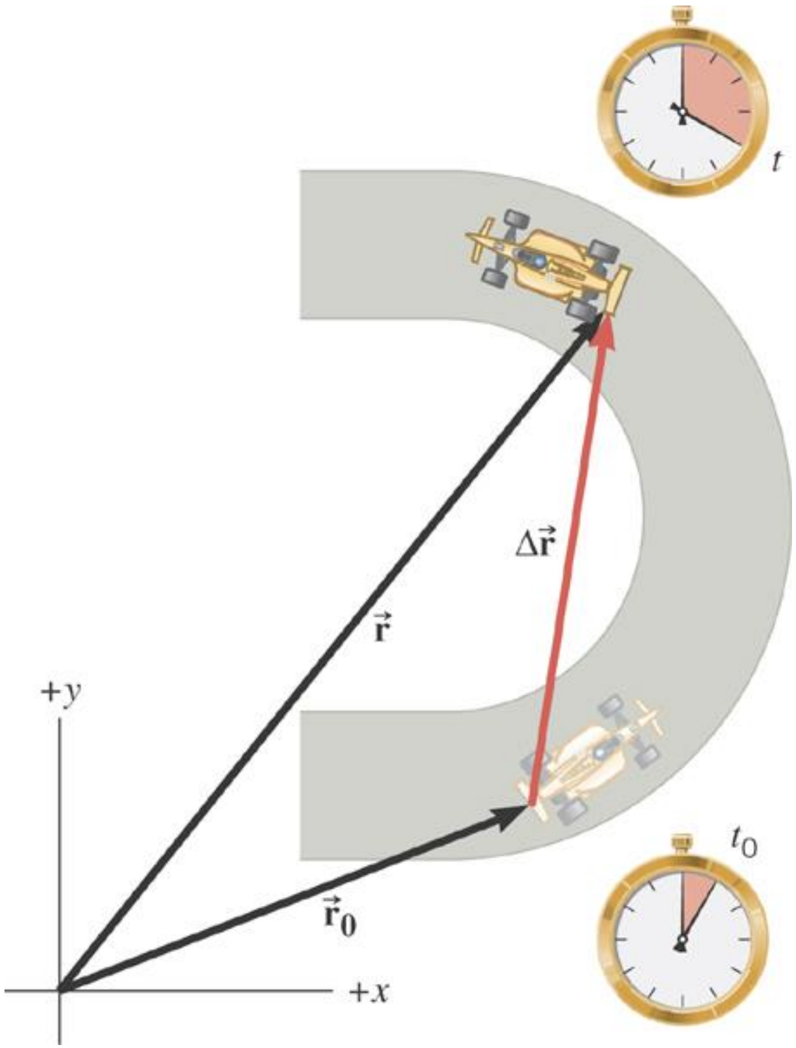
Average velocity is the displacement divided by the elapsed time.

$$\bar{\vec{\mathbf{v}}} = \frac{\vec{\mathbf{r}} - \vec{\mathbf{r}}_o}{t - t_o} = \frac{\Delta\vec{\mathbf{r}}}{\Delta t}$$



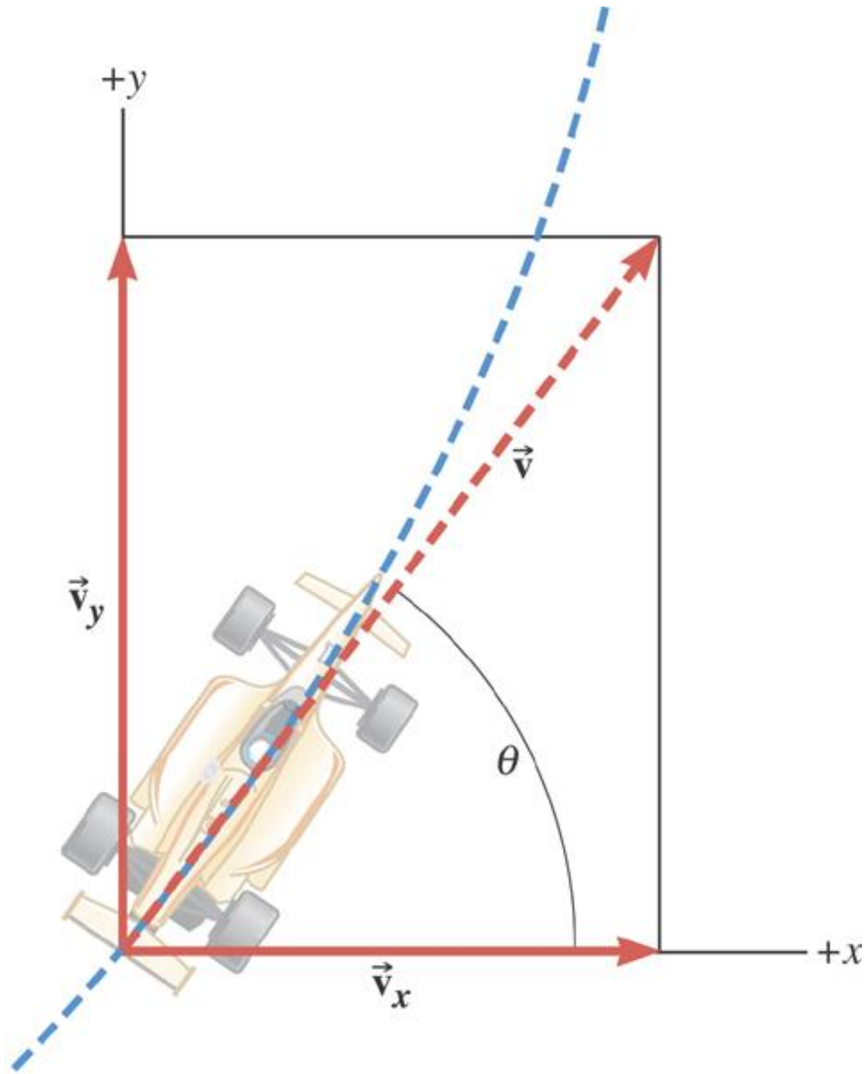
The ***instantaneous velocity*** indicates how fast the car moves and the direction of motion at each instant of time.

$$\overline{\vec{\mathbf{v}}} = \lim_{\Delta t \to 0} \frac{\Delta \vec{\mathbf{r}}}{\Delta t}$$



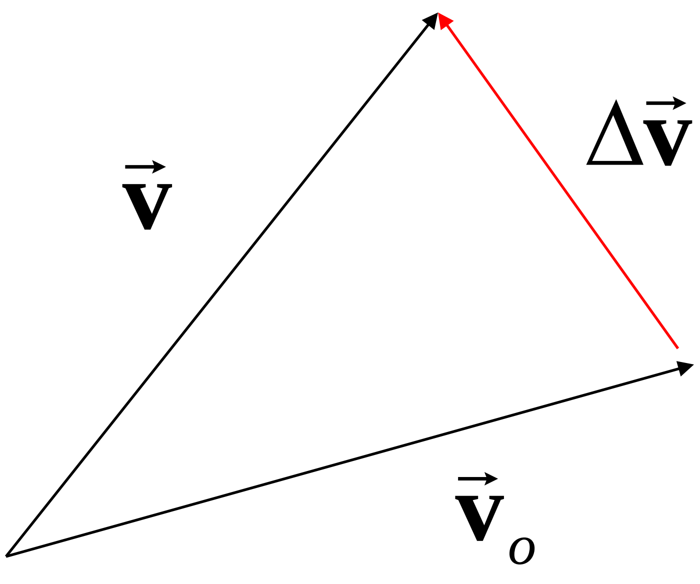
2.1 Displacement, Velocity, and Acceleration

$$\overline{\vec{\mathbf{v}}} = \lim_{\Delta t \to 0} \frac{\Delta \vec{\mathbf{r}}}{\Delta t} = \frac{dr}{dt}$$



DEFINITION OF AVERAGE ACCELERATION

$$\bar{\vec{\mathbf{a}}} = \frac{\vec{\mathbf{v}} - \vec{\mathbf{v}}_o}{t - t_o} = \frac{\Delta \vec{\mathbf{v}}}{\Delta t}$$



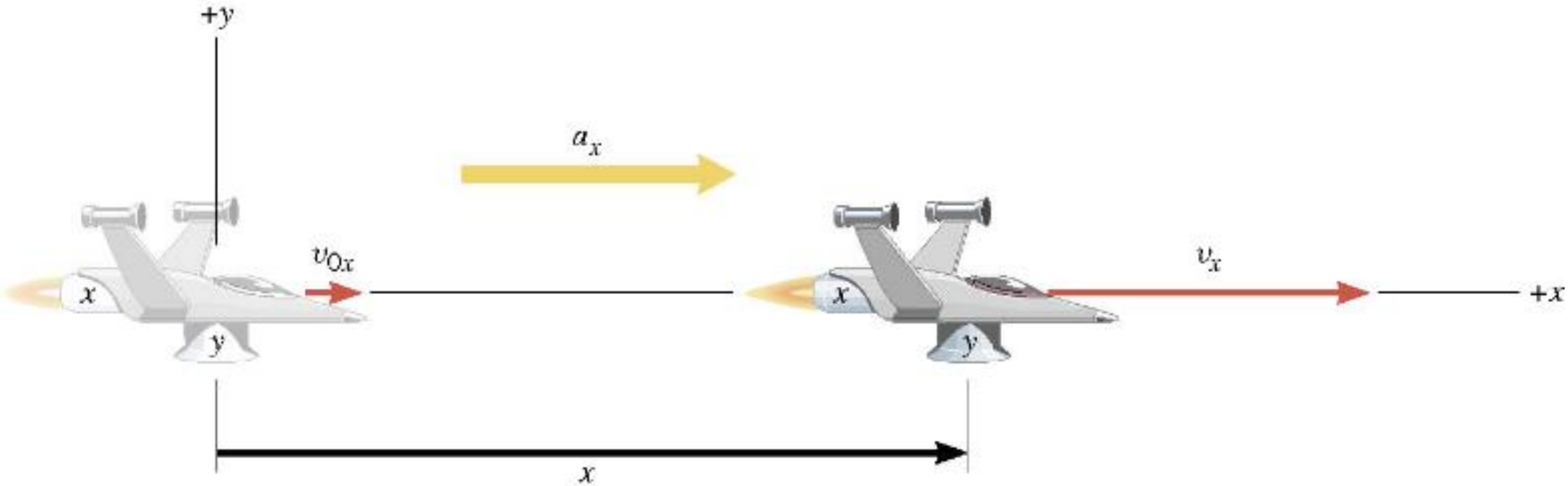
Equations of Kinematics

$$v = v_o + at$$

$$v^2 = v_o^2 + 2ax$$

$$x = v_o t + \tfrac{1}{2} a t^2$$

2.2 Equations of Kinematics in Two Dimensions

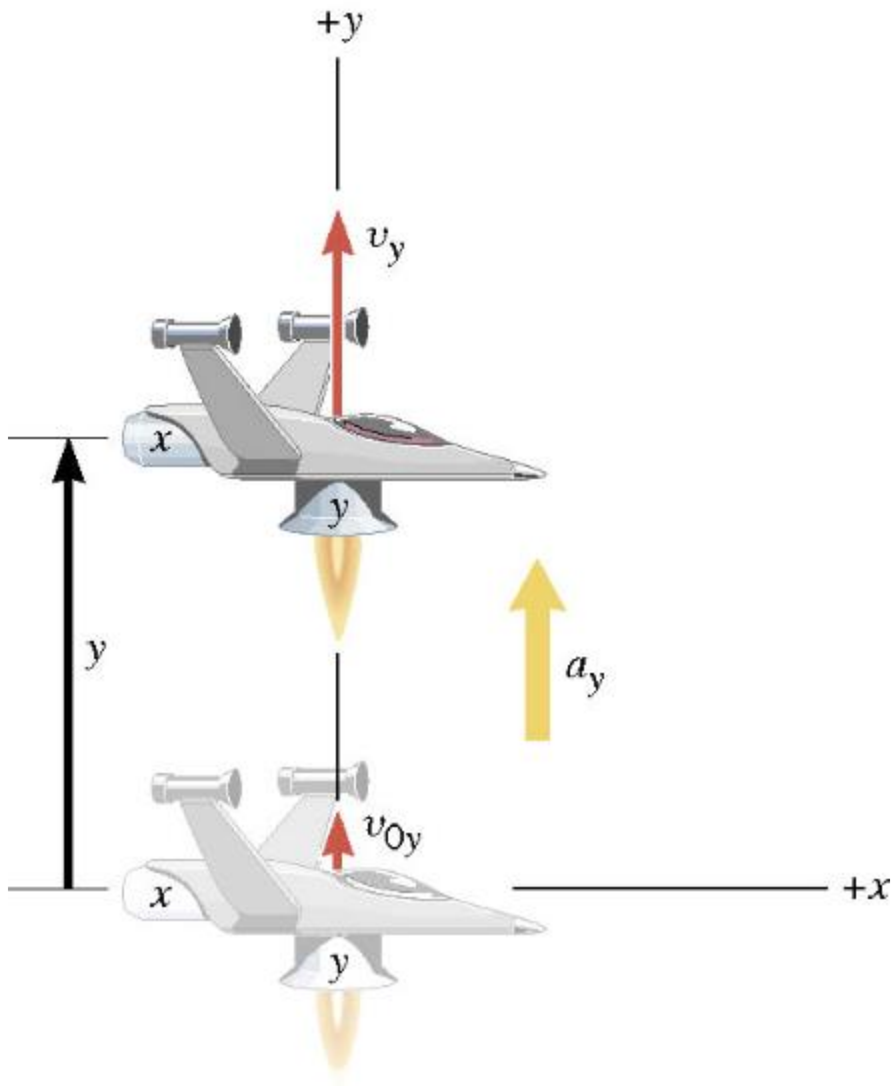


$$v_x = v_{ox} + a_x t$$

$$v_x^2 = v_{ox}^2 + 2a_x x$$

$$x = v_{ox}t + \tfrac{1}{2}a_x t^2$$

2.2 Equations of Kinematics in Two Dimensions

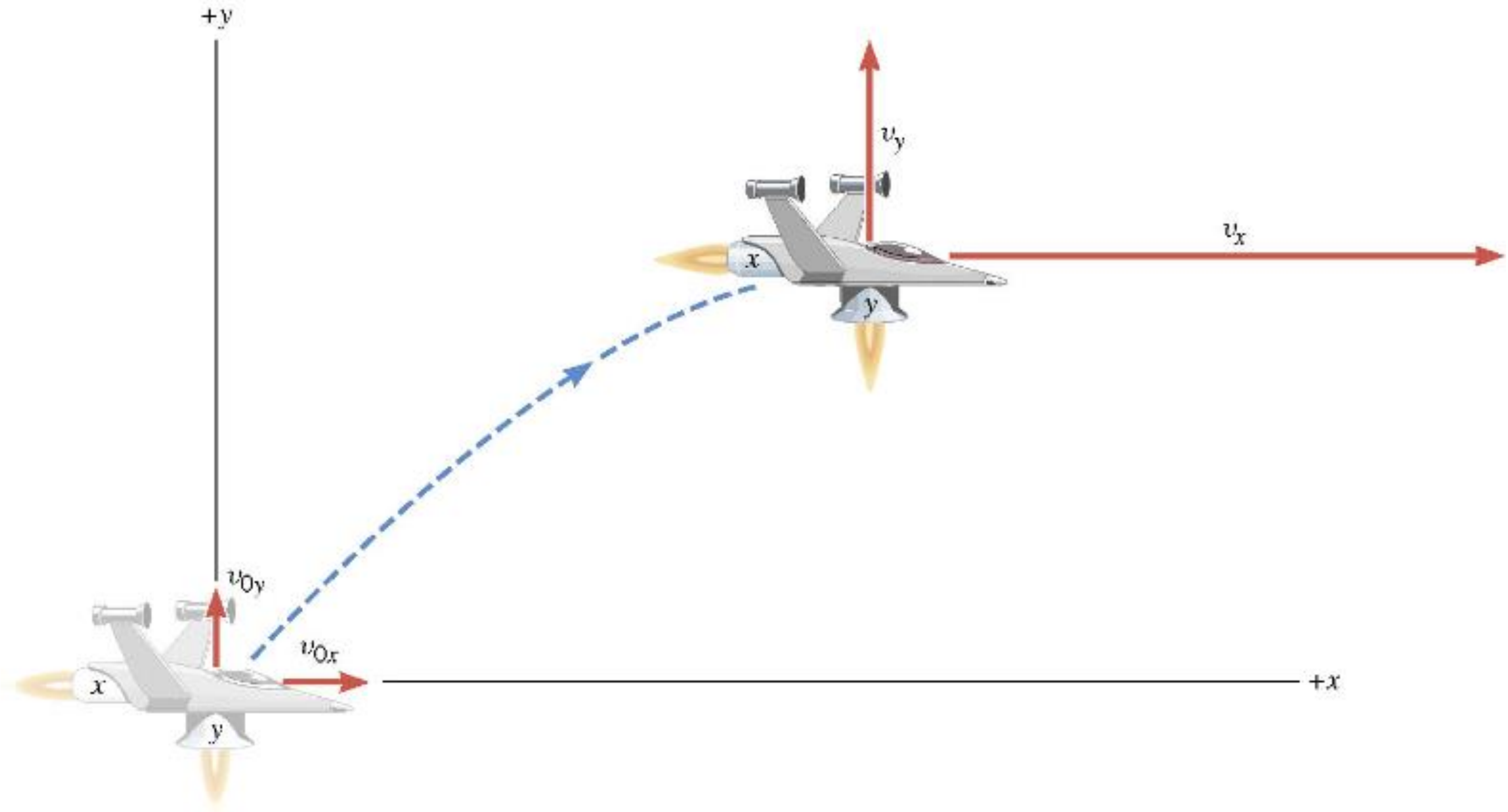


$$v_y = v_{oy} + gt$$

$$y = v_{oy}t + \tfrac{1}{2}gt^2$$

$$v_y^2 = v_{oy}^2 + 2gy$$

2.2 Equations of Kinematics in Two Dimensions



The x part of the motion occurs exactly as it would if the y part did not occur at all, and vice versa.

Reasoning Strategy

1. Make a drawing.

2. Decide which directions are to be called positive (+) and negative (-).

3. Write down the values that are given for any of the five kinematic variables associated with each direction.

4. Verify that the information contains values for at least three of the kinematic variables. Do this for *x* and *y*. Select the appropriate equation.

5. When the motion is divided into segments, remember that the final velocity of one segment is the initial velocity for the next.

6. Keep in mind that there may be two possible answers to a kinematics problem.

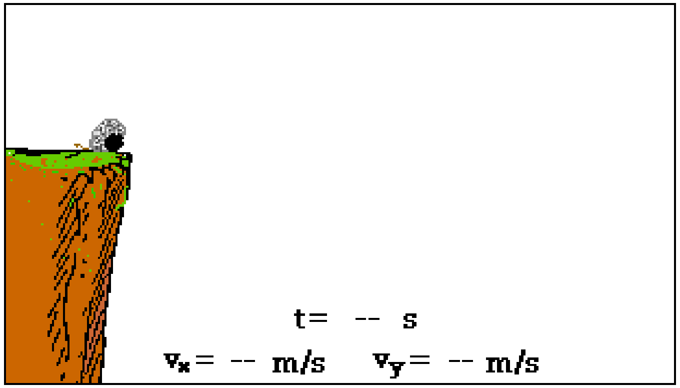
2.3 Projectile Motion

Under the influence of gravity alone, an object near the surface of the Earth will accelerate downwards at 9.80m/s^2.

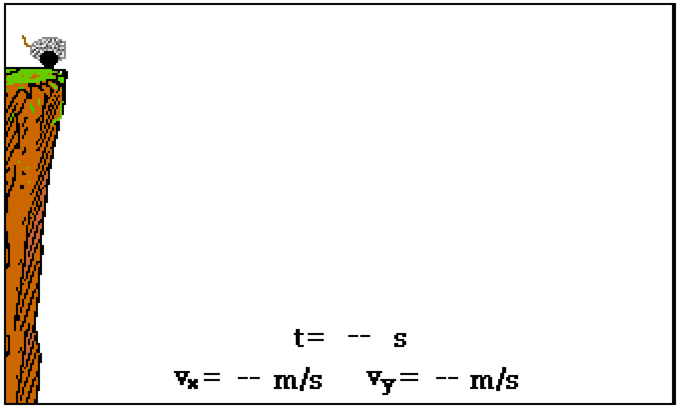
$$g = -9.80\,\mathrm{m/s^2} \qquad a_x = 0$$

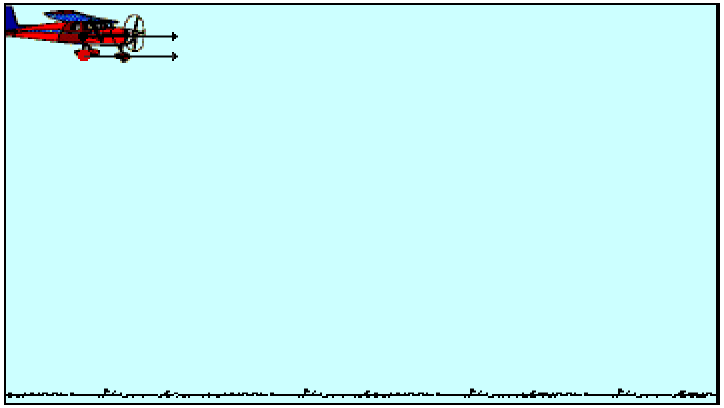
$$v_x = v_{ox} = \text{constant}$$

Projectile



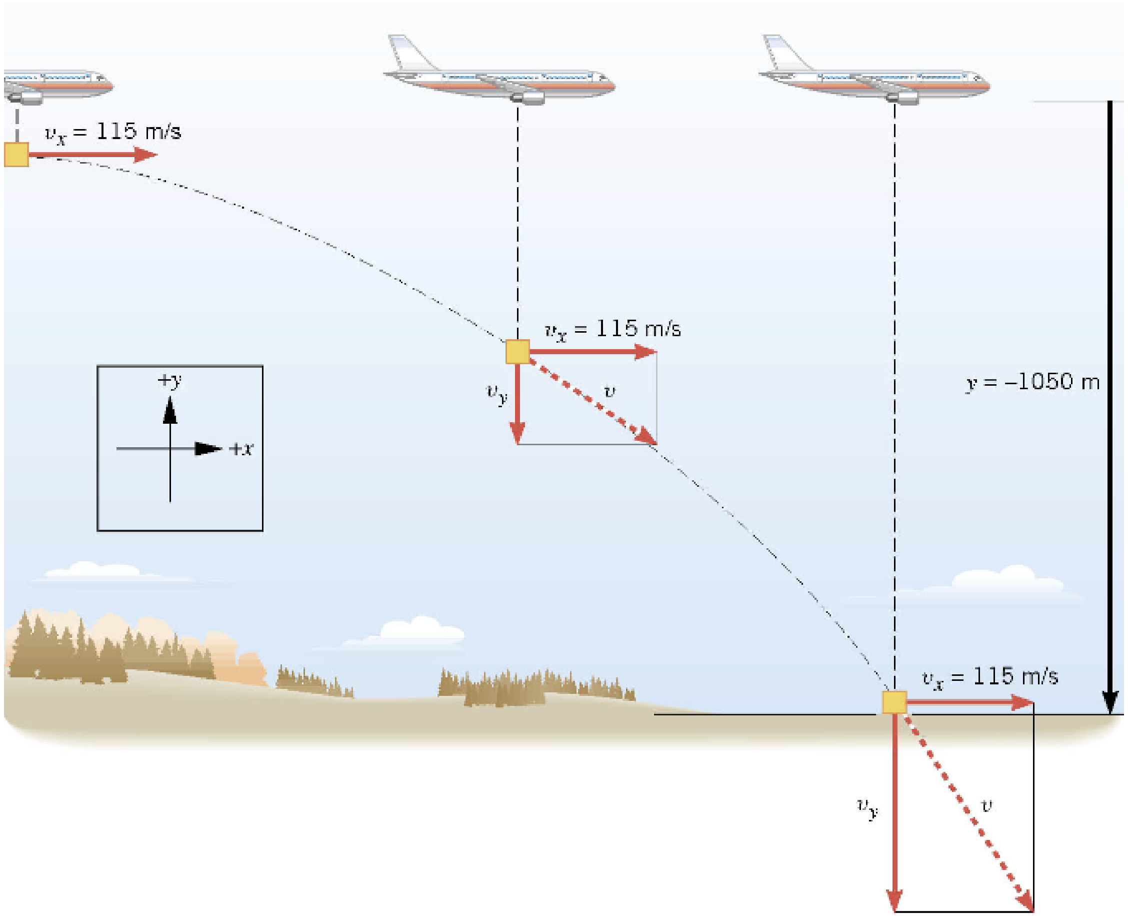
Horizontally Launched Projectile



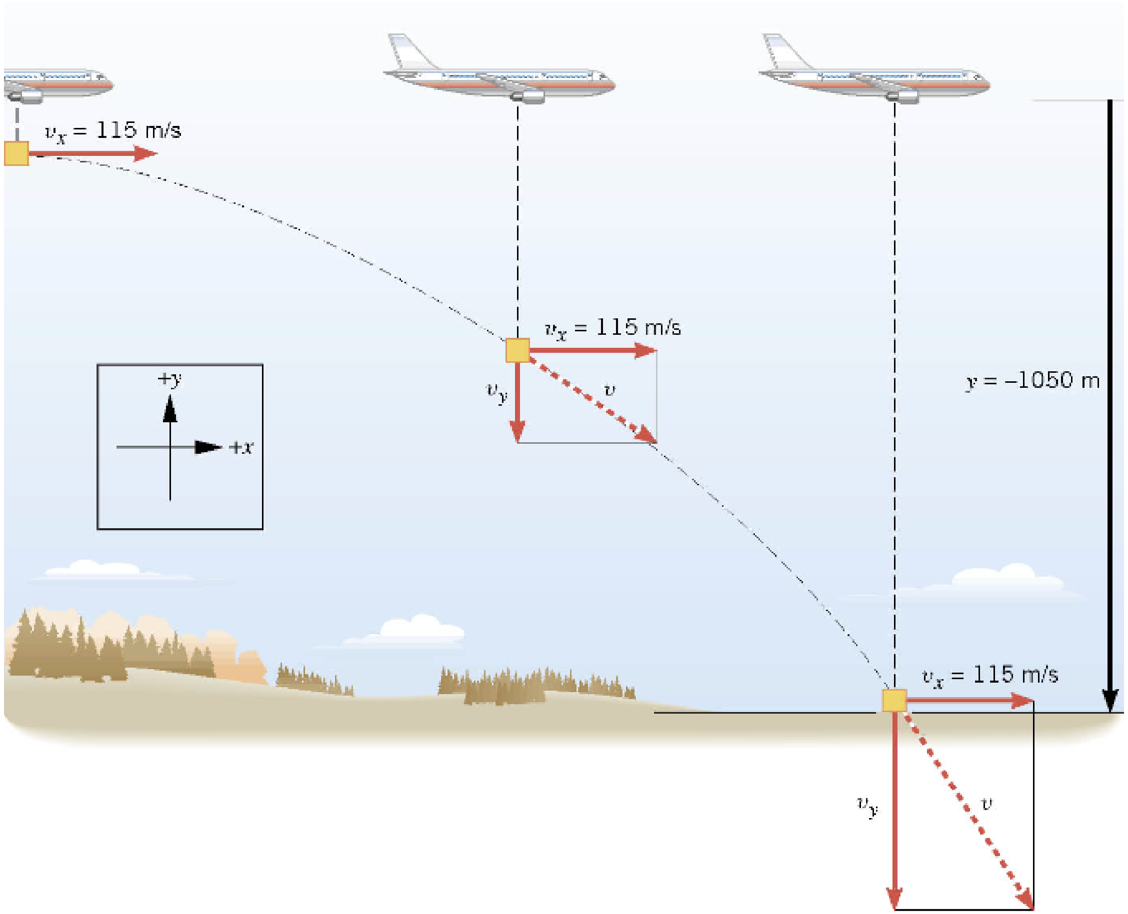


Example 3 A Falling Care Package

The airplane is moving horizontally with a constant velocity of +115 m/s at an altitude of 1050m. Determine the time required for the care package to hit the ground.



2.3 Projectile Motion



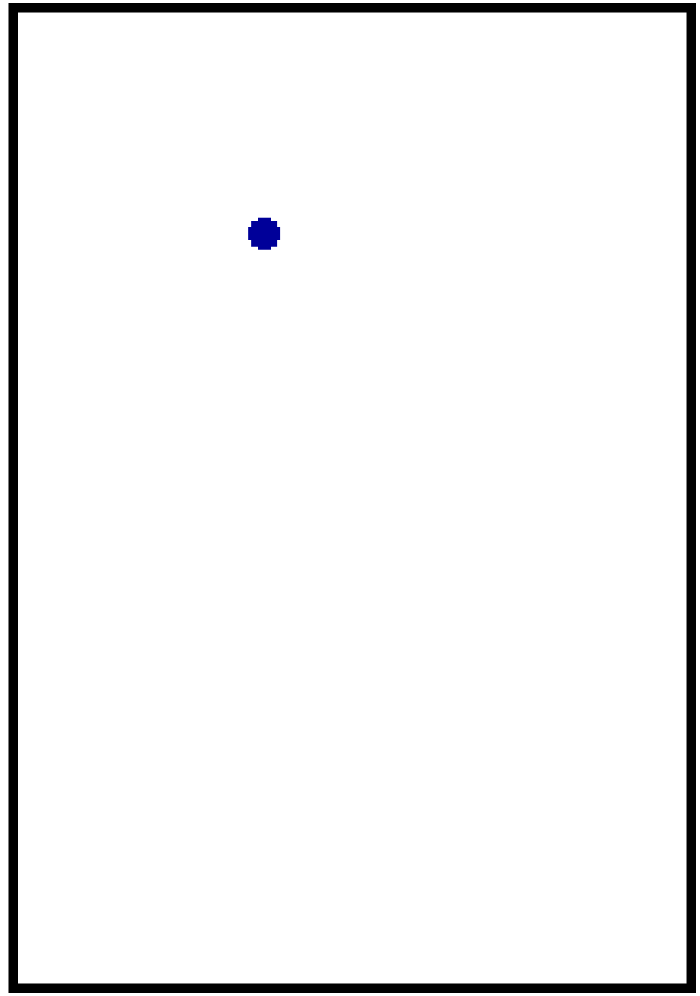
y	g	v_y	v_{oy}	t
-1050 m	-9.80 m/s^2		0 m/s	?

y	g	v_y	v_{oy}	t
-1050 m	-9.80 m/s^2		0 m/s	?

$$y = v_{oy}t + \tfrac{1}{2}gt^2 \quad \Longrightarrow \quad y = \tfrac{1}{2}gt^2$$

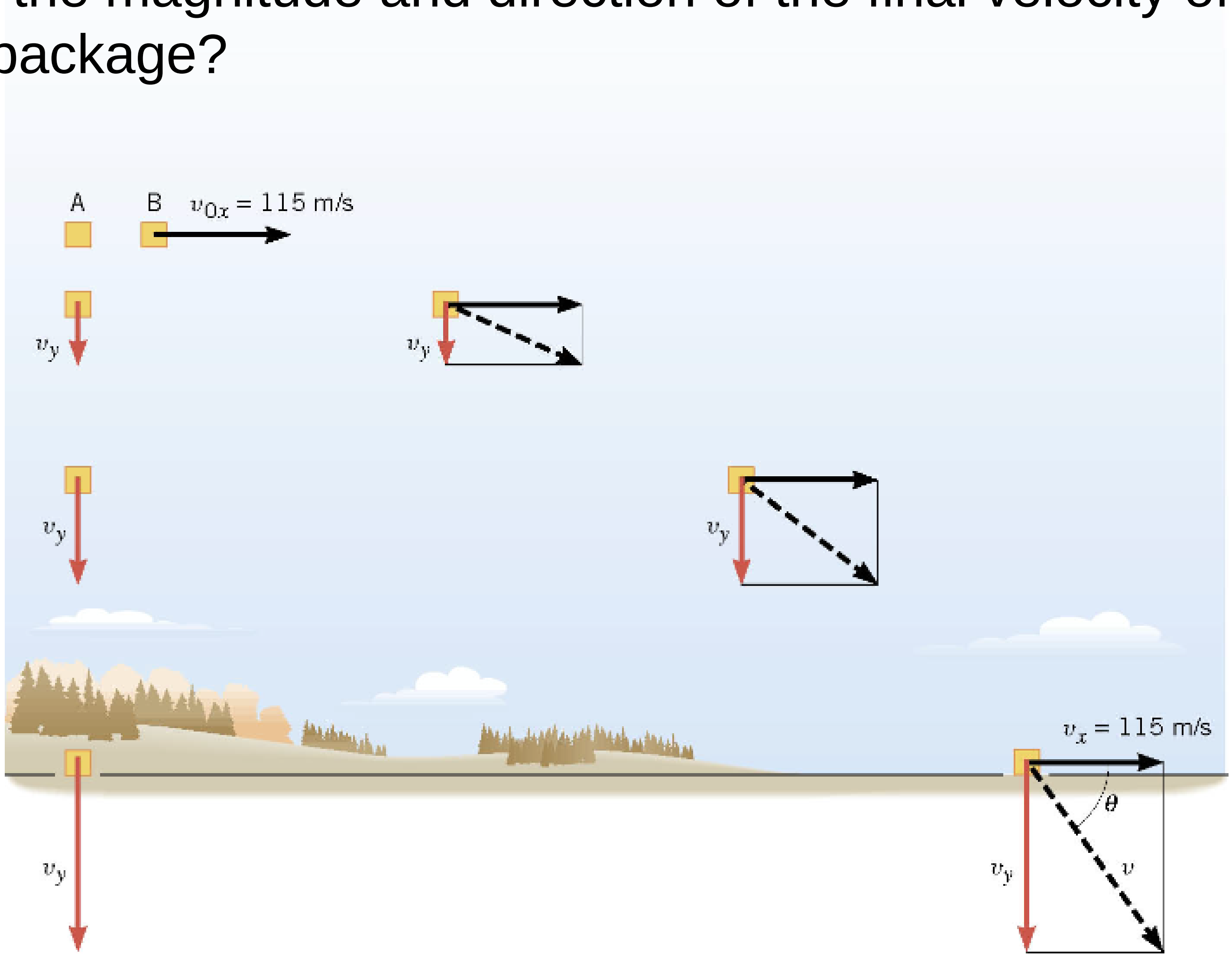
$$t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2(-1050\,\mathrm{m})}{-9.80\,\mathrm{m/s^2}}} = 14.6\,\mathrm{s}$$

Parabolic Motion of Projectiles

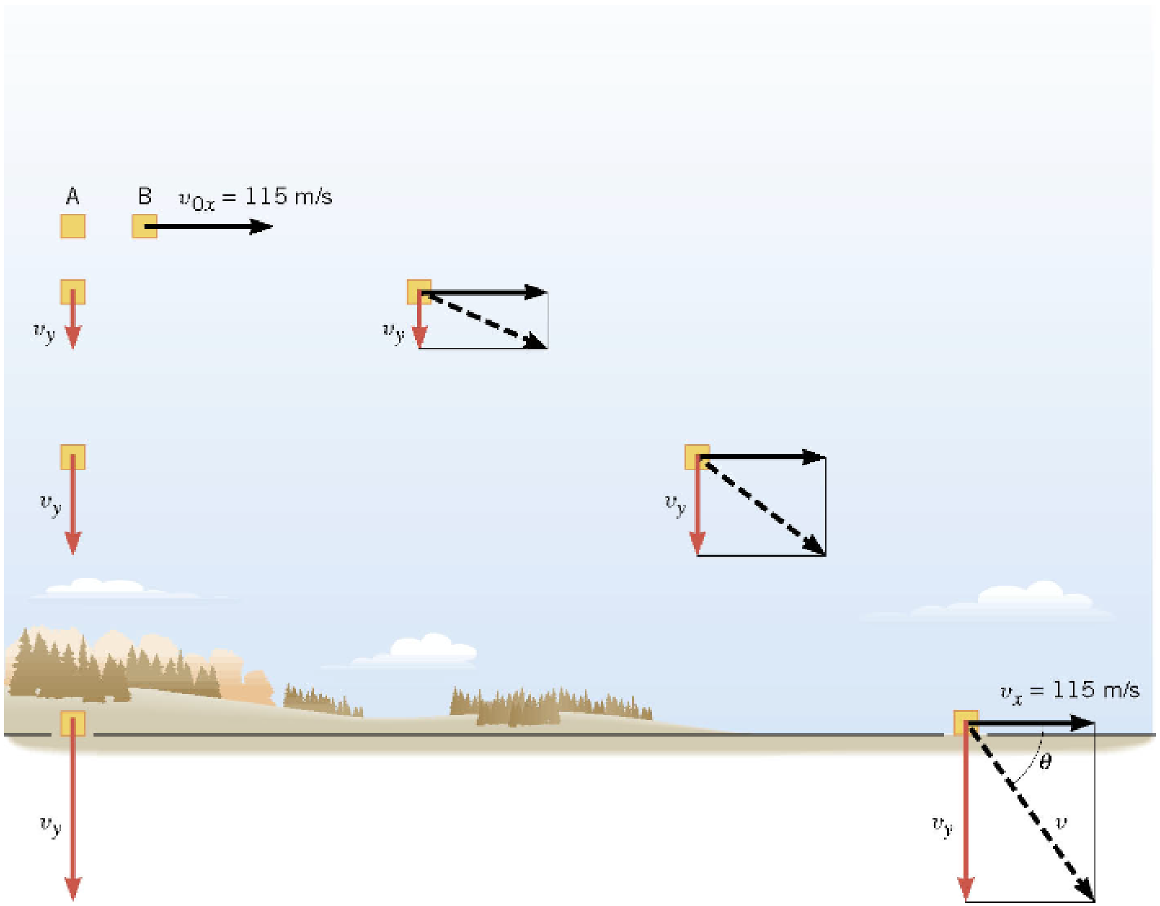


Example 4 The Velocity of the Care Package

What are the magnitude and direction of the final velocity of the care package?



2.3 Projectile Motion



y	g	v_y	v_{oy}	t
-1050 m	-9.80 m/s^2	?	0 m/s	14.6 s

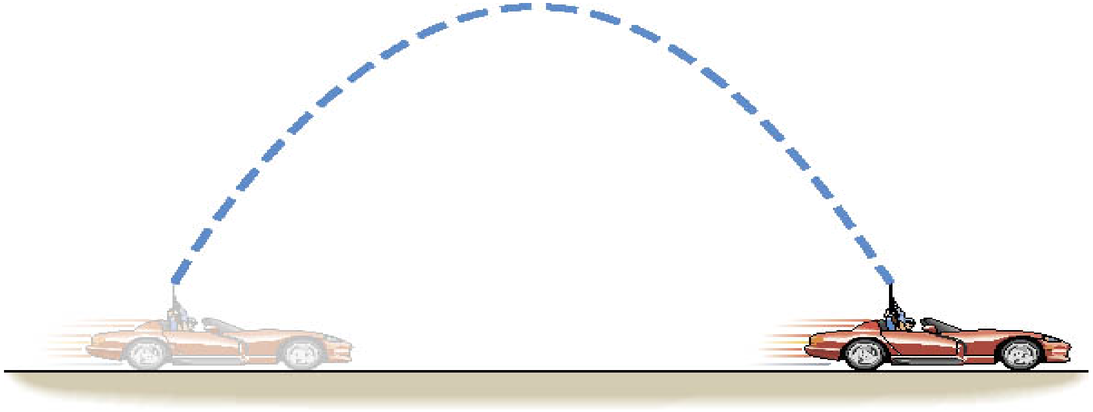
y	g	v_y	v_{oy}	t
-1050 m	-9.80 m/s^2	?	0 m/s	14.6 s

$$v_y = v_{oy} + gt = 0 + \left(-9.80\,\mathrm{m/s^2}\right)\left(14.6\,\mathrm{s}\right)$$

$$= -143\,\mathrm{m/s}$$

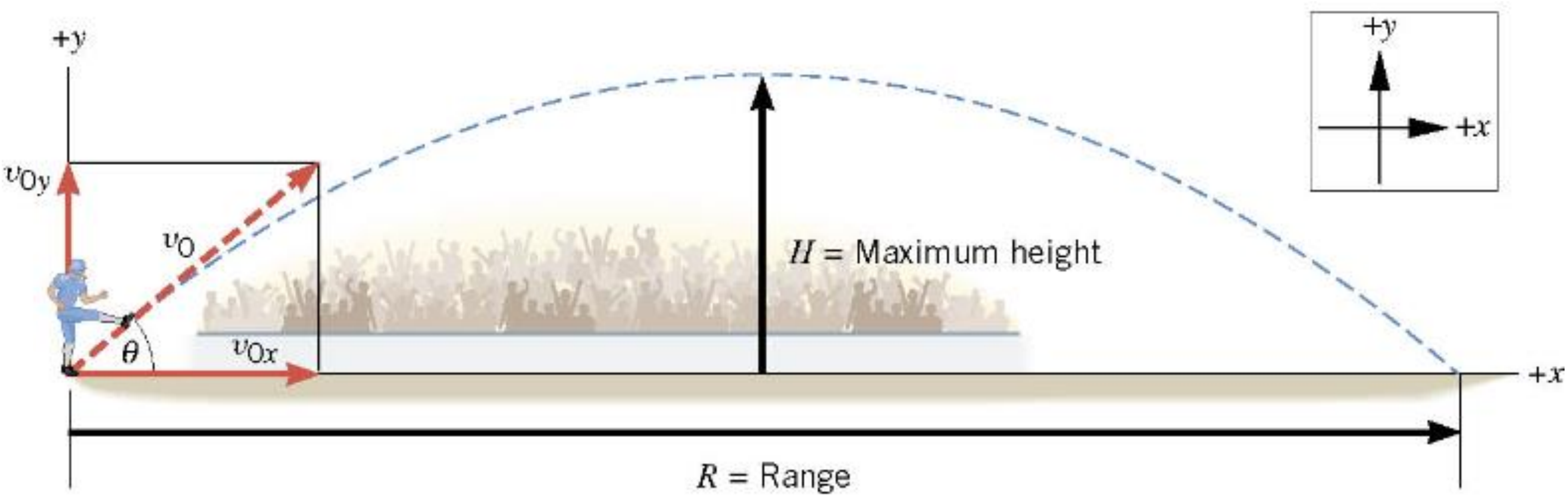
Conceptual Example 5 I Shot a Bullet into the Air...

Suppose you are driving a convertible with the top down. The car is moving to the right at constant velocity. You point a rifle straight up into the air and fire it. In the absence of air resistance, where would the bullet land – behind you, ahead of you, or in the barrel of the rifle?

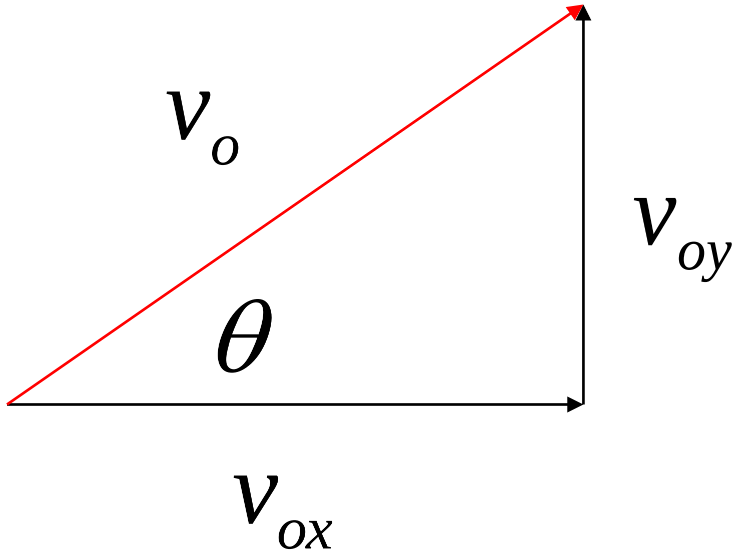


Example 6 The Height of a Kickoff

A placekicker kicks a football at and angle of 40.0 degrees and the initial speed of the ball is 22 m/s. Ignoring air resistance, determine the maximum height that the ball attains.



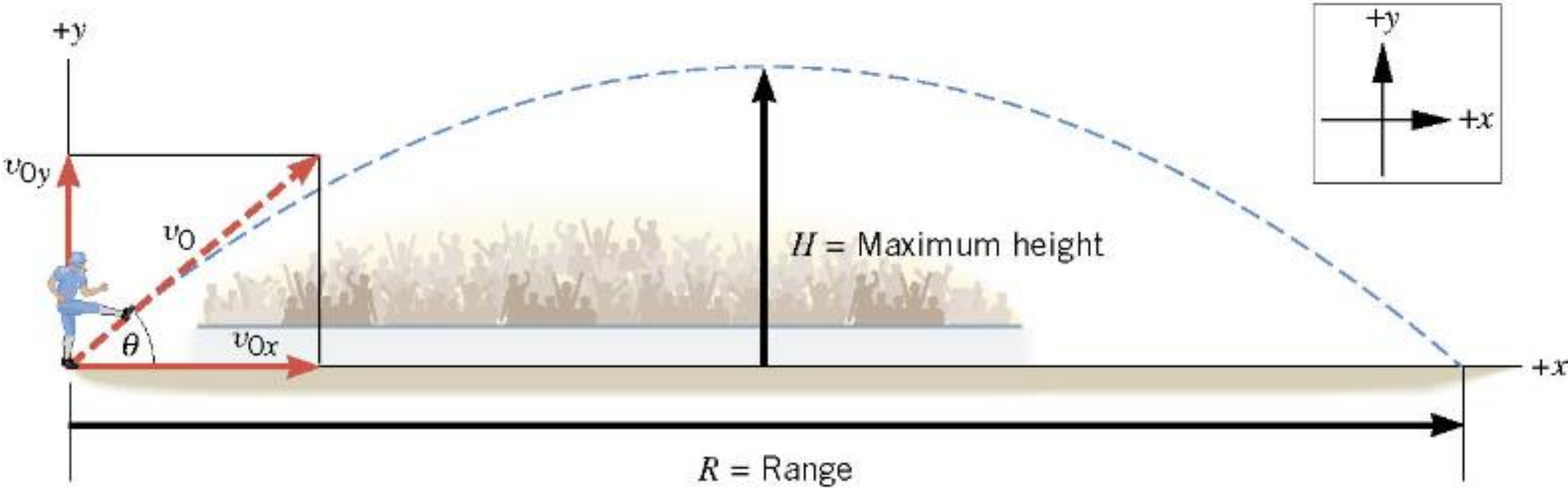
2.3 Projectile Motion



$$v_{oy} = v_o \sin\theta = \left(22\,\mathrm{m/s}\right)\sin 40^\circ = 14\,\mathrm{m/s}$$

$$v_{ox} = v_o \sin\theta = \left(22\,\mathrm{m/s}\right)\cos 40^\circ = 17\,\mathrm{m/s}$$

2.3 Projectile Motion



y	g	v_y	v_{oy}	t
?	-9.80 m/s^2	0	14 m/s	

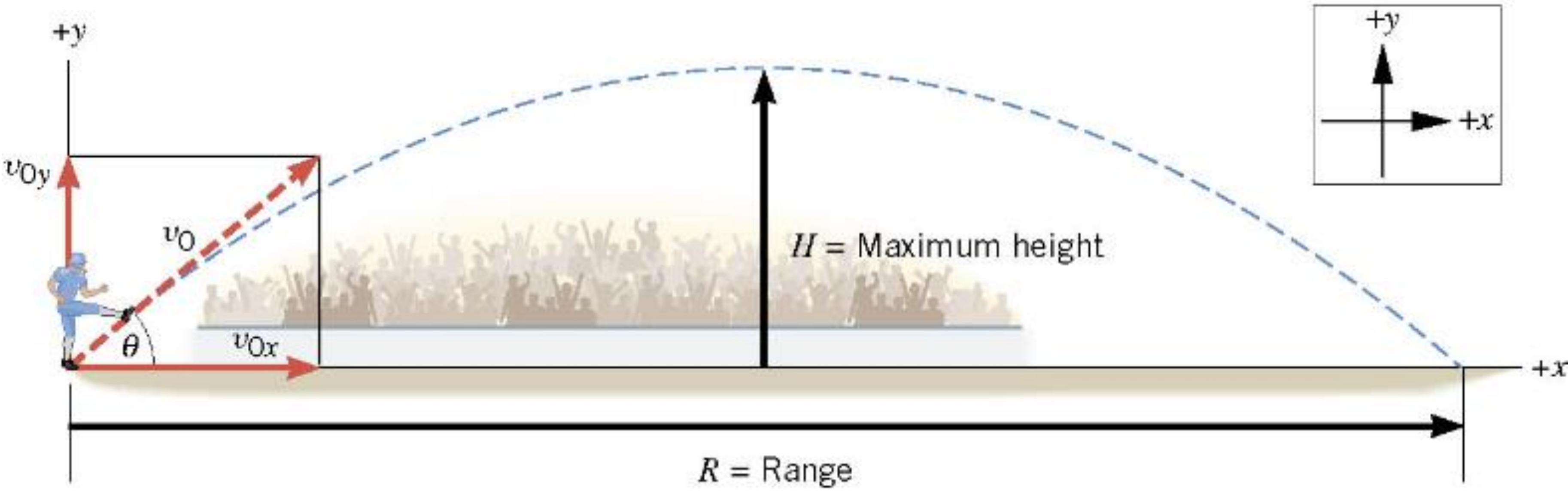
y	g	v_y	v_{oy}	t
?	-9.80 m/s^2	0	14 m/s	

$$v_y^2 = v_{oy}^2 + 2gy \quad \Longrightarrow \quad y = \frac{v_y^2 - v_{oy}^2}{2g}$$

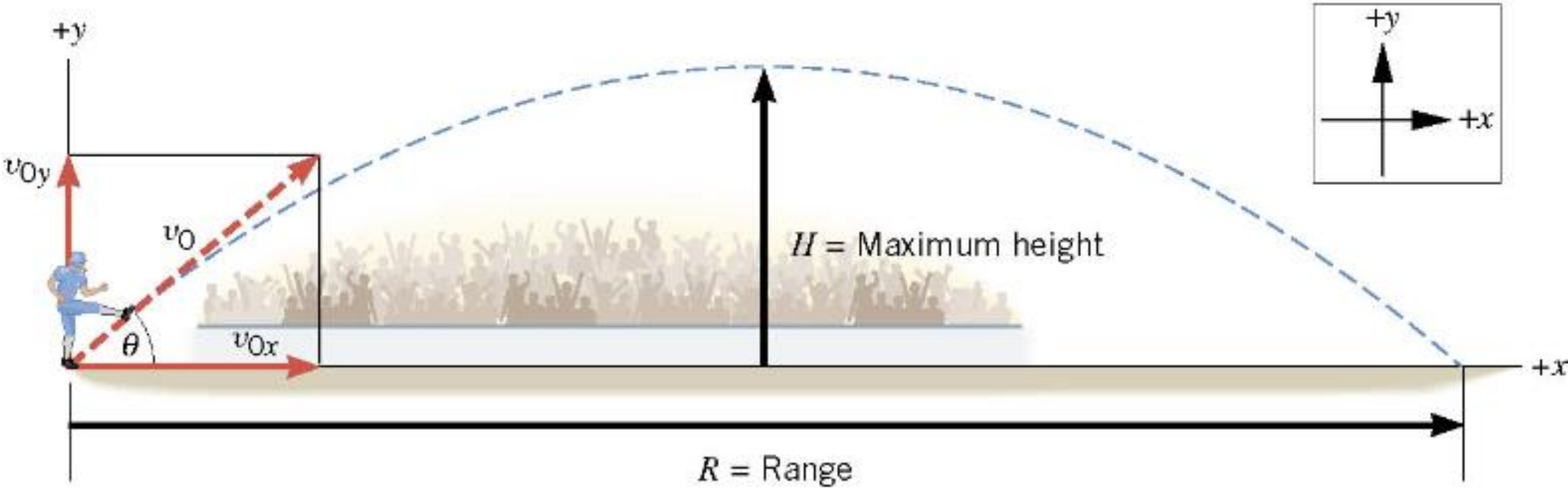
$$y = \frac{0 - (14\,\mathrm{m/s})^2}{2(-9.8\,\mathrm{m/s^2})} = +10\,\mathrm{m}$$

Example 7 The Time of Flight of a Kickoff

What is the time of flight between kickoff and landing?



2.3 Projectile Motion



y	g	v_y	v_{oy}	t
0	-9.80 m/s^2		14 m/s	?

y	g	v_y	v_{oy}	t
0	-9.80 m/s^2		14 m/s	?

$$y = v_{oy}t + \tfrac{1}{2}gt^2$$

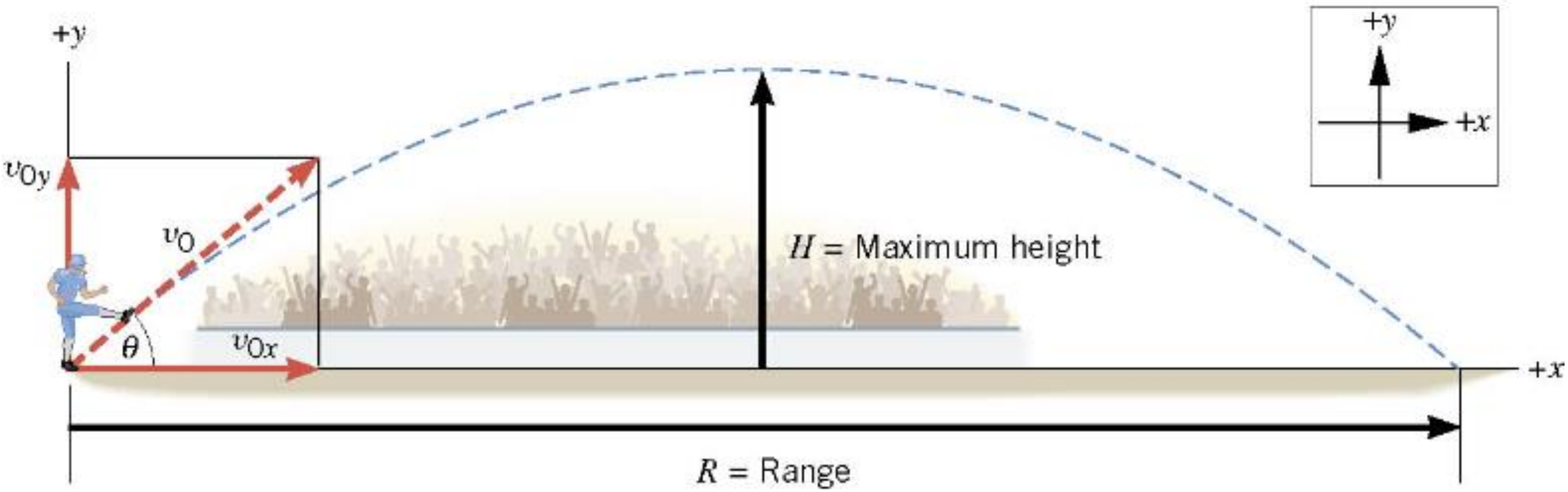
$$0 = (14\,\mathrm{m/s})t + \tfrac{1}{2}\left(-9.80\,\mathrm{m/s^2}\right)t^2$$

$$0 = 2(14\,\mathrm{m/s}) + \left(-9.80\,\mathrm{m/s^2}\right)t$$

$$t = 2.9\,\mathrm{s}$$

Example 8 The Range of a Kickoff

Calculate the range R of the projectile.



$$x = v_{ox}t + \tfrac{1}{2}a_x t^2 = v_{ox}t$$

$$= (17\,\mathrm{m/s})(2.9\,\mathrm{s}) = +49\,\mathrm{m}$$

Maximum height, H

- The ball reaches the highest point at point B at velocity, v where
  - x-component of the velocity, $v_x = v = vo\cos\theta$
  - y-component of the velocity, $v_y = 0$
  - y-component of the displacement, $y = H$
- Use $v_y^2 = v_{oy}^2 - 2gy$

$$0 = (vo\sin\theta)^2 - 2gH$$

$$\boxed{H = \frac{vo^2 \sin^2\theta}{2g}}$$

Horizontal range, R

$$\boxed{R = \frac{vo^2 \sin 2\theta}{g}}$$

Time taken to reach maximum height

$$\boxed{\Delta t = \frac{2vo\sin\theta}{g}}$$