

Class Discussion

Math Concepts

The sum of 7 and 5 times of a number is 48

$$7 + 5m = 48$$

3 less than 4 times of a number is 5

$$4m - 3 = 5$$

What is 30 percent of 600 ?

$$m = 30\% \times 600$$

$$m = \frac{30}{100} \times 600$$

$$m = 180$$

What percent of 500 is 75 ?

$$\frac{m}{100} \times 500 = 75$$

$$\frac{m}{100} \times 500 = 75$$

$$5m = 75$$

$$m = 15$$

Kinematics- Examples

x

$$v = \frac{dx}{dt}$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

Differentiation

$$x = \int_{t_1}^{t_2} v dt$$

$$v = \int_{t_1}^{t_2} a dt$$

a

Integration

Example-1

- An airplane accelerates down a runway at 3.20 m/s^2 for 32.8 s until it finally lifts off the ground. Determine the distance traveled before takeoff.
- Given:
- $a = +3.2 \text{ m/s}^2$ $t = 32.8 \text{ s}$ $v_i = 0 \text{ m/s}$
- Find: $d = ??$

$$y = v_i t + \frac{1}{2} a t^2$$

- $d = (0 \text{ m/s}) * (32.8 \text{ s}) + 0.5 * (3.20 \text{ m/s}^2) * (32.8 \text{ s})^2$
- $d = 1720 \text{ m}$

Example-2

- A stone is dropped into a deep well and is heard to hit the water 3.41 s after being dropped. Determine the depth of the well.
- Given:
- $a = -9.8 \text{ m/s}^2$ $t = 3.41 \text{ s}$ $v_i = 0 \text{ m/s}$
- Find: $d = ??$

$$y = v_i t + \frac{1}{2} a t^2$$

- $d = (0 \text{ m/s}) * (3.41 \text{ s}) + 0.5 * (-9.8 \text{ m/s}^2) * (3.41 \text{ s})^2$
- $d = 0 \text{ m} + 0.5 * (-9.8 \text{ m/s}^2) * (11.63 \text{ s}^2)$
- $d = -57.0 \text{ m}$

Position and Displacement

- In one dimension

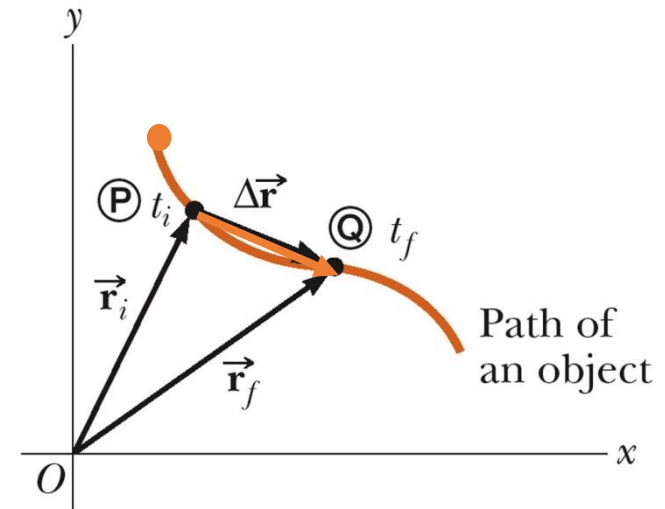
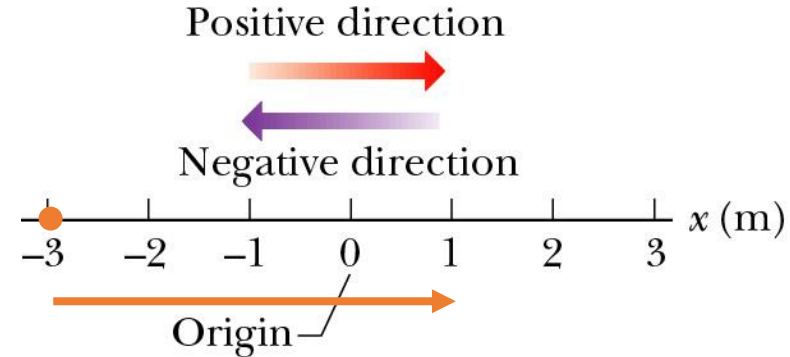
$$\Delta x = x_2(t_2) - x_1(t_1)$$

- In two dimensions

- Displacement:

$$\Delta \vec{r} = \vec{r}_2 - \vec{r}_1$$

$$\begin{aligned}\Delta \vec{r} &= (x_2 \hat{i} + y_2 \hat{j}) - (x_1 \hat{i} + y_1 \hat{j}) \\ &= (x_2 - x_1) \hat{i} + (y_2 - y_1) \hat{j} \\ &= \Delta x \hat{i} + \Delta y \hat{j}\end{aligned}$$



Average & Instantaneous Velocity

- Average velocity

$$\vec{v}_{avg} \equiv \frac{\Delta \vec{r}}{\Delta t}$$

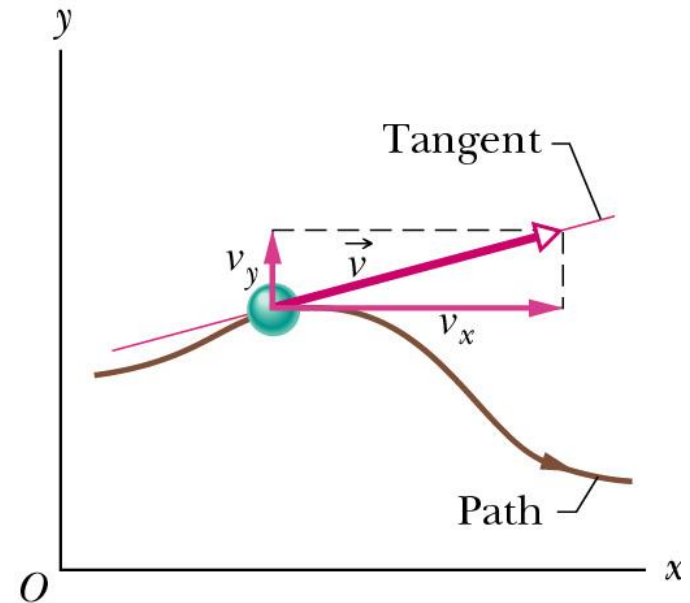
$$\vec{v}_{avg} = \frac{\Delta x}{\Delta t} \hat{i} + \frac{\Delta y}{\Delta t} \hat{j} = v_{avg,x} \hat{i} + v_{avg,y} \hat{j}$$

- Instantaneous velocity

$$\vec{v} \equiv \lim_{t \rightarrow 0} \vec{v}_{avg} = \lim_{t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt} \hat{i} + \frac{dy}{dt} \hat{j} = v_x \hat{i} + v_y \hat{j}$$

- v is tangent to the path in x-y graph;



Average & Instantaneous Acceleration

- Average acceleration

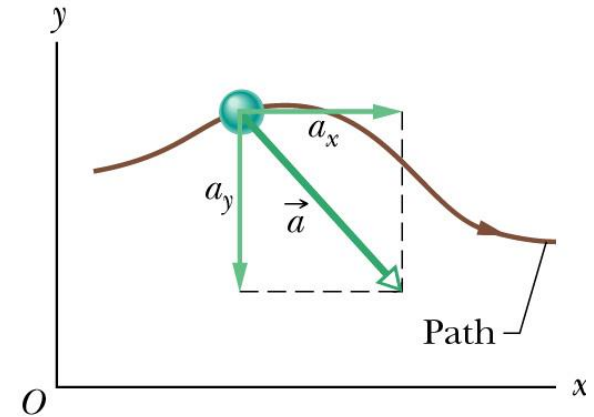
$$\vec{a}_{avg} \equiv \frac{\Delta \vec{v}}{\Delta t}$$

- Instantaneous acceleration

$$\vec{a} \equiv \lim_{t \rightarrow 0} \vec{a}_{avg} = \lim_{t \rightarrow 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt} \hat{i} + \frac{dv_y}{dt} \hat{j} = a_x \hat{i} + a_y \hat{j}$$

- Both the magnitude and the direction can change



Summary in two dimension

□ Position $\vec{r}(t) = x\hat{i} + y\hat{j}$

□ Average velocity $\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} = v_{avg,x}\hat{i} + v_{avg,y}\hat{j}$

□ Instantaneous velocity $v_x \equiv \frac{dx}{dt}$ $v_y \equiv \frac{dy}{dt}$

$$\vec{v}(t) = \lim_{t \rightarrow 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = v_x\hat{i} + v_y\hat{j}$$

□ Acceleration $a_x \equiv \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$ $a_y \equiv \frac{dv_y}{dt} = \frac{d^2y}{dt^2}$

$$\vec{a}(t) = \lim_{t \rightarrow 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} = a_x\hat{i} + a_y\hat{j}$$

Motion in 2 Dims- Constant acceleration

- Motions in each dimension are independent components
- Constant acceleration equations

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

$$\vec{r} = \vec{v}_0 t + \frac{1}{2} \vec{a} t^2$$

$$v^2 = v_0^2 + 2a r$$

- Constant acceleration equations hold in **each** dimension

$$v_x = v_{0x} + a_x t$$

$$v_y = v_{0y} + a_y t$$

$$x = v_{0x} t + \frac{1}{2} a_x t^2$$

$$y = v_{0y} t + \frac{1}{2} a_y t^2$$

$$v_x^2 = v_{0x}^2 + 2a_x(x)$$

$$v_y^2 = v_{0y}^2 + 2a_y(y)$$

Projectile Motion- Example

- A “projectile” is an object on which the only influence is gravity.
 - Usually they are thrown/fired/etc. but are **not** things like rockets,
 - Always, gravity will accelerate the object down.
 - Horizontal motion + Vertical motion
 - Horizontal: $a_x = 0$, constant velocity motion $v_x = v_{0x}$
 - Vertical: $a_y = -g = -9.8 \text{ m/s}^2$, $v_{0y} = 0$
 - Equations:

Horizontal

$$v_x = v_{0x} + a_x t$$

$$x = v_{0x} t + \frac{1}{2} a_x t^2$$

$$v_x^2 = v_{0x}^2 + 2a_x(x)$$

Zero

Zero

Zero

Vertical

$$v_y = v_{0y} + a_y t$$

$$y = v_{0y} t + \frac{1}{2} a_y t^2$$

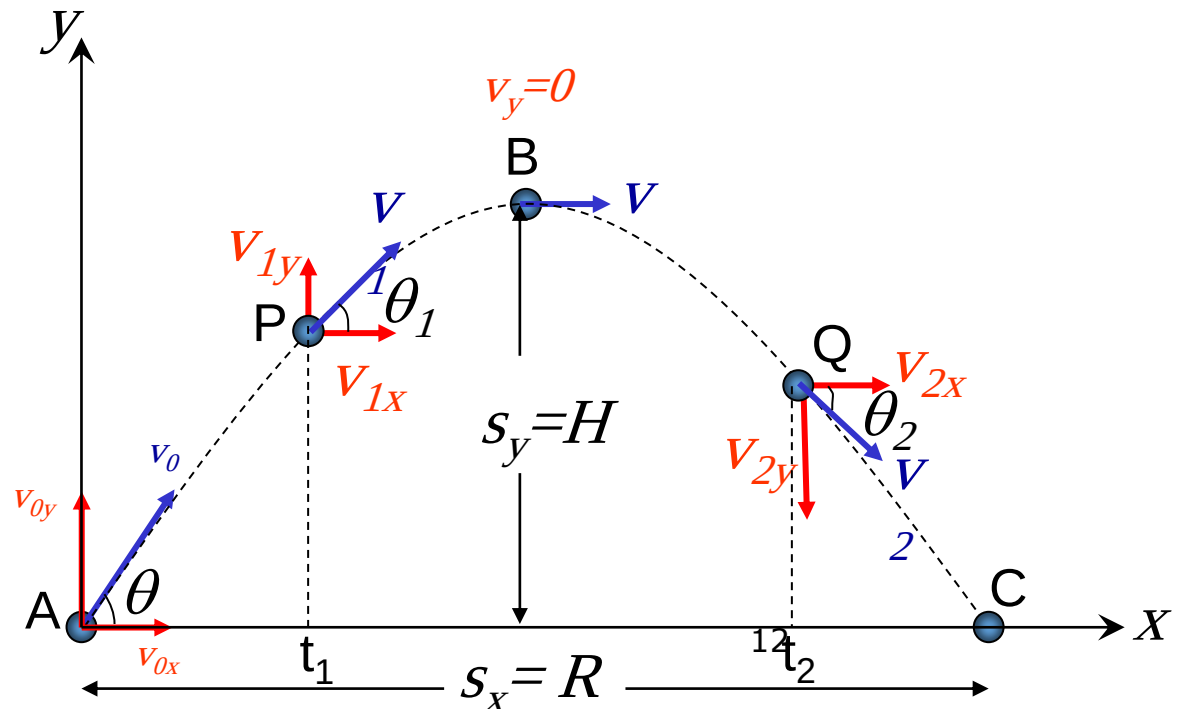
$$v_y^2 = v_{0y}^2 + 2a_y(y)$$

Projectile motion

- The path followed by a projectile is called trajectory

$$v_x = v_0 \cos \theta$$

$$v_y = v_0 \sin \theta$$



What is R and h ?

Time of fly

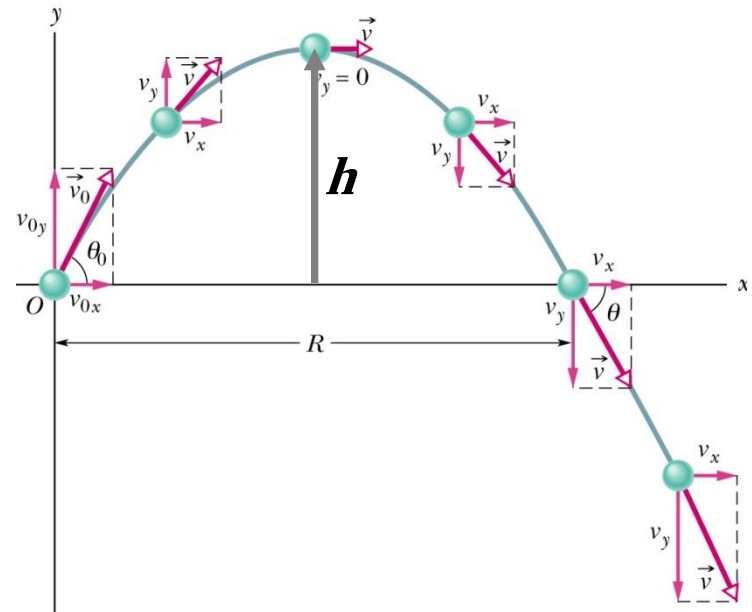
$$t = \frac{2v_0 \sin \theta_0}{g}$$

Range: max horizontal distance

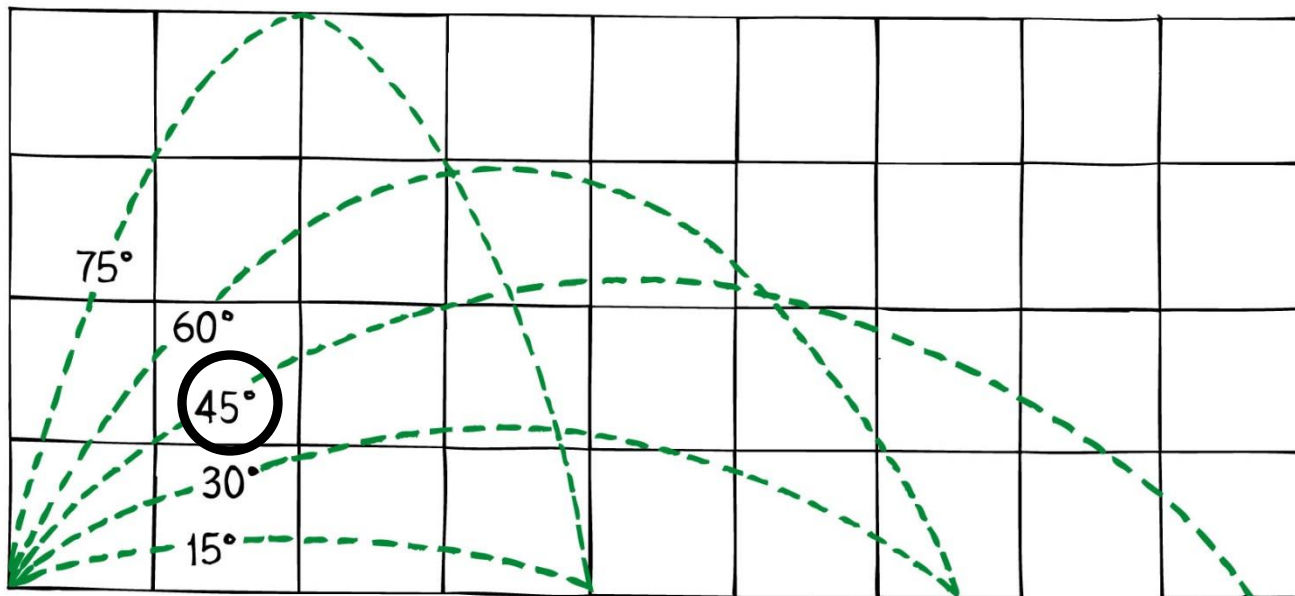
$$R = \frac{v_0^2 \sin 2\theta_0}{g}$$

Height: max Vertical distance

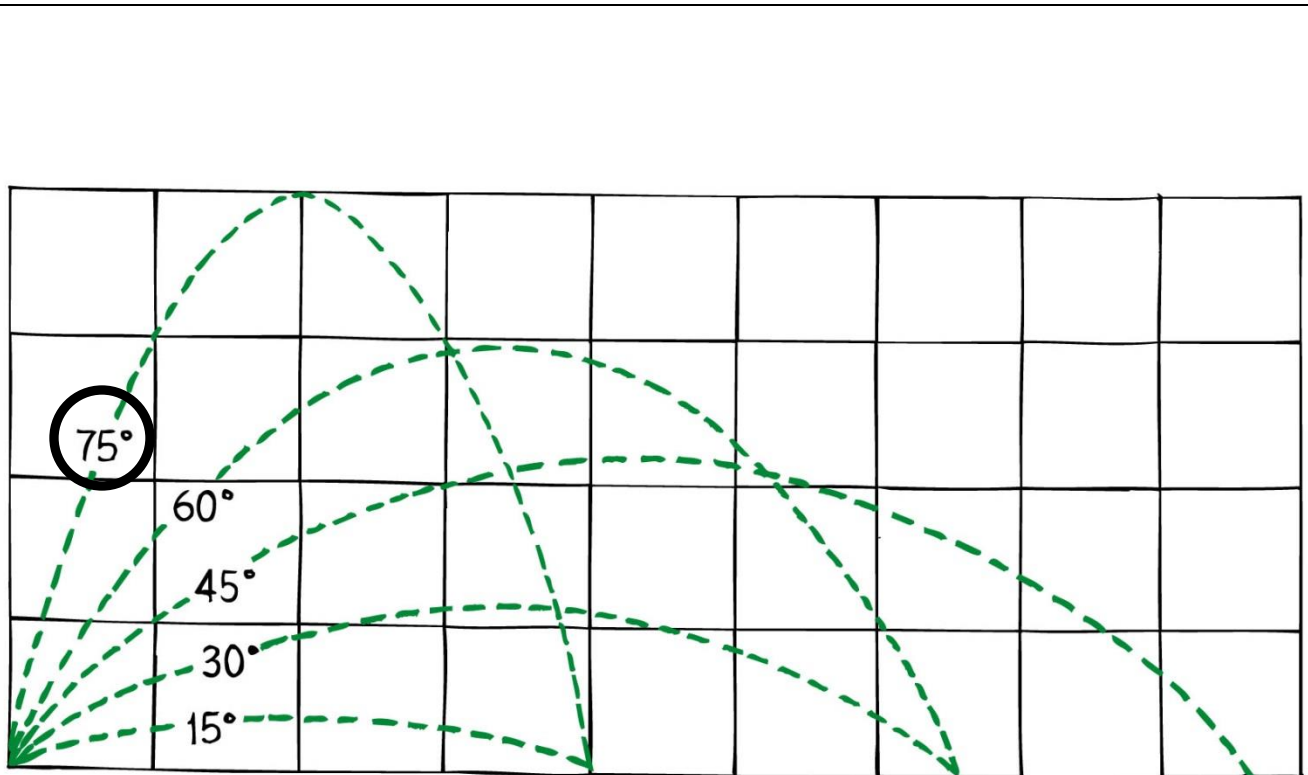
$$h = \frac{v_0^2 \sin^2 \theta_0}{2g}$$



For an object fired at the same velocity in all cases, with which angle does the projectile go farthest?

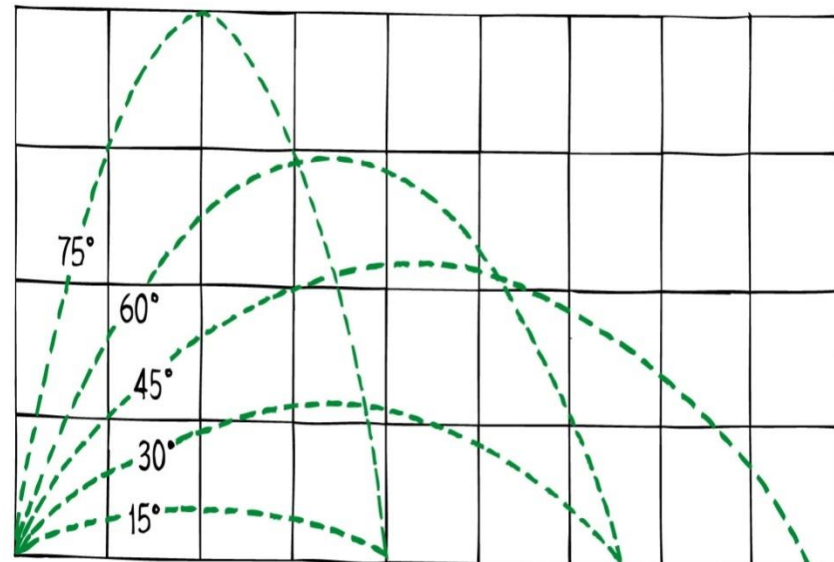


For an object fired at the same velocity in all cases, with which angle does the projectile go highest?



So all that means...

- 45° Is the angle at which a projectile will go the farthest. (best *range*)
- The higher the object goes up, the longer it is in the air.
(**more height = more time**)
- The greater the angle, the greater the hang time.



Example-3

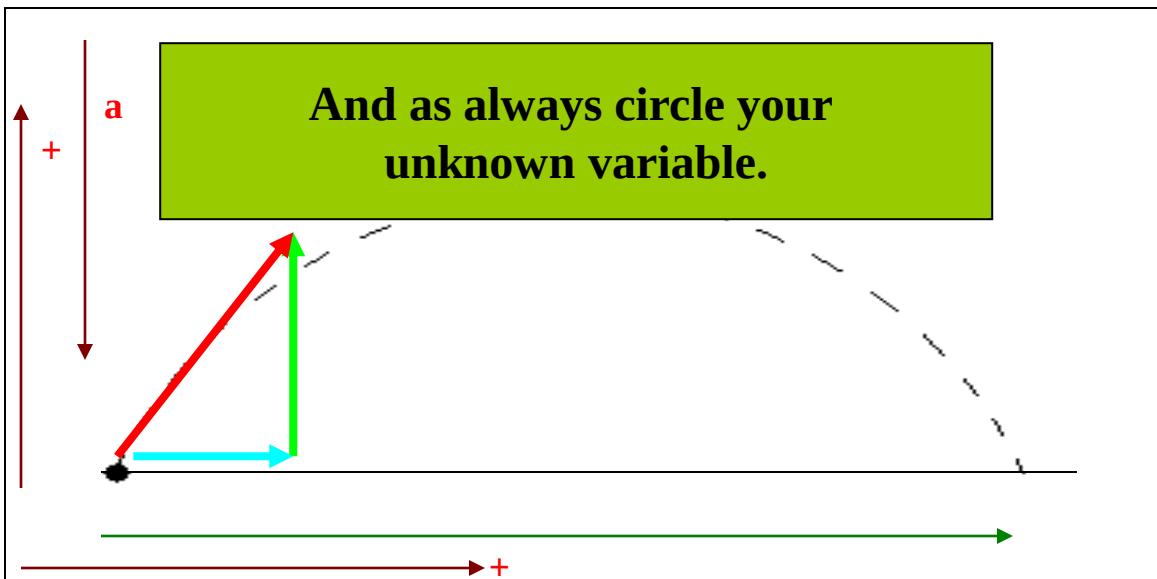
A soccer ball is kicked at an angle of 70° from the horizontal with a velocity of 20m/s . What is the *range* of the soccer ball? (how far away does it land)

As always, start by drawing a diagram, including all vectors.

We need to use trig functions to solve for v_{ix} and v_{iy}

Begin to fill in what you can on your givens table

And as always circle your unknown variable.



x	y
$v_o = 6.84\text{m/s}$	$v_o = 18.8\text{m/s}$
$v = 6.84\text{m/s}$	$v = 18.8\text{m/s}$
$a_x = 0\text{ m/s}^2$	$a_y = -10.0\text{ m/s}^2$
$t = ?$	$t = ?$
$x = ?$	$y = 0\text{ m}$

$$v_{ox} = V \cos \Theta$$

$$v_{ox} = (20) \cos(70)$$

$$v_{ox} = 6.84\text{ m/s}$$

$$v_{oy} = V \sin \Theta$$

$$v_{oy} = (20) \sin(70)$$

$$v_{oy} = 18.8\text{ m/s}$$

Example-3- Cont.

A soccer ball is kicked at an angle of 70° from the horizontal with a velocity of 20.0m/s . What is the *range* of the soccer ball? (how far away does it land)

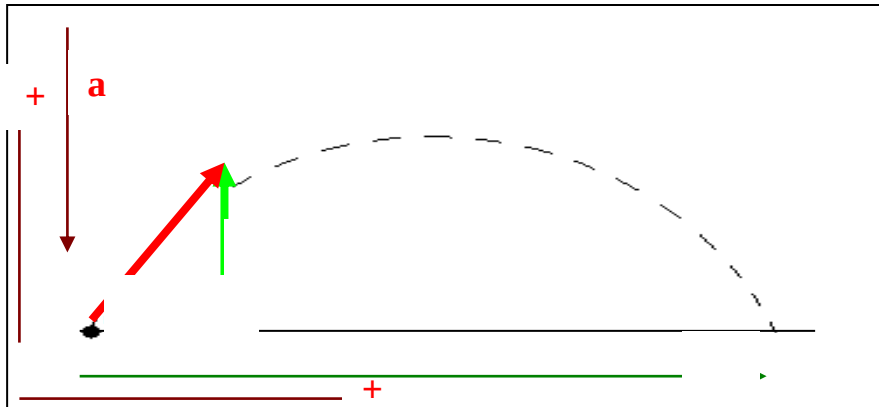
x	y
$v_o = 6.84 \text{ m/s}$	$v_o = +18.8 \text{ m/s}$
$v = 6.84 \text{ m/s}$	$v = -18.8 \text{ m/s}$
$a_x = 0 \text{ m/s}^2$	$a_y = -10.0 \text{ m/s}^2$
$t = ?$	$t = ?$
$x = ?$	$y = 0 \text{ m}$

$$x = v_{ox}t + \frac{1}{2}at^2$$

$$x = (6.84)(\underline{t}) + \frac{1}{2}(0)\underline{t}^2$$

$$x = (6.84)(3.76) + 0$$

$$x = 25.7 \text{ m}$$



$$v = v_o + at$$

$$t = (v - v_o) / a$$

$$t = (-18.8 - 18.8) / (-10.0)$$

$$t = \underline{\underline{3.76 \text{ s}}}$$

H.W

A bottle dropped from the window of a bus moving at 60 kmh^{-1} . If the window is 196 cm . High, find the distance along the track which the stone moves before striking the ground.



Force and Motion

State of Newton's Law

- Newton's first law explains the motion of an object when only balanced forces ($F_{\text{net}} = 0$, this means $\Sigma F = 0$) are acting.
- Newton's second law explains the motion of an object when unbalanced forces ($F_{\text{net}} \neq 0$, this means $\Sigma F = ma$) are acting.

Whenever one body exerts a force on a second body, the second body exerts an oppositely directed force of equal magnitude on the first body. ($F_{12} = -F_{21}$)

- **Example 4**

- If the force is given as 2.0 N and the mass is 500 g, what acceleration will be given to the mass?

- 4.0 m/s²

- Calculating the weight of an object is just a special case of Newton's second law. You will use the acceleration due to gravity as the acceleration in the equation.
- The weight vector is always towards the center of the earth since that is the direction of the acceleration due to gravity.
- **Weight** is the gravitational force exerted by a large body (such as Earth). $F_w = mg$. The acceleration of gravity (g) on Earth is 9.8 m/s^2 .

Example 5

- Astronauts left a 500 kg rover on the moon where the acceleration due to gravity is 1.6 m/s^2 .
 - What is the mass of the rover on the moon?
 - ----- kg
 - What is the weight of the rover on the moon?
 - ----- N
 - What was the weight of the rover on the earth?
 - ----- N

Example 6

- A body moves with constant speed in a straight line. Which of the following statements must be true?
 - A. No force acts on the body
 - B. A single constant force acts on the body in the direction of motion
 - C. A single constant force acts on the body in the direction opposite to the motion.
 - D. A net force of zero acts on the body
 - E. A constant net force acts on the body in the direction of motion

Example 7

- A force accelerates a body of mass M . The same force applied to a second body produces three times the acceleration. What is the mass of the second body?
 - A. M
 - B. $3M$
 - C. $M/3$
 - D. $9M$
 - E. $M/9$

Example 8

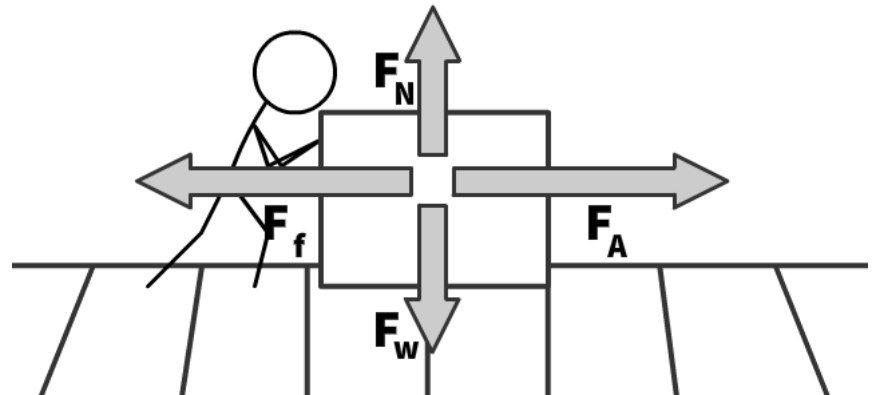
- A net force of 64 N acts on a mass of 16 kg. The resulting acceleration is
 - A. 16 m/s^2
 - B. 0.51 m/s^2
 - C. 64 m/s^2
 - D. 9.0 m/s^2
 - E. 4.0 m/s^2

Example 9

- A mass of 25 kg is acted on by two forces. F_1 is 15 N due East, and F_2 is 10 N due North. The acceleration of the mass is
 - A. 0.72 m/s^2 ; 56.3° north of east
 - B. 0.20 m/s^2 ; due east
 - C. 0.72 m/s^2 ; 33.7° north of east
 - D. 1.0 m/s^2 ; 33.7° north of east
 - E. 0.20 m/s^2 ; 56.3° north of east

Draw free-body diagrams, identifying external forces such as weight, normal forces, frictional forces, tension, etc.; use these diagrams in the solution of dynamics problems.

- A free-body diagram which shows all the external forces vectors acting on a body as lines with arrows and the object is symbolized by a dot.
- If there are multiple objects making up a system, you will need to draw a free body diagram for the system as well as each individual object making up the system.



Non-fundamental Forces

- A **normal force** is the force exerted on a body by a surface against which it is pressed.
 - The normal force is **usually** perpendicular to the surface.
- A **frictional force** is the force on a body when the body slides or attempts to slide along a surface.
 - The force is parallel to the surface and directed so as to oppose the motion of the body.
 - When the body is in motion, you have kinetic friction
 - When the body is not in motion, you have static friction.

Example 10

- A 150 kg football player stands on a bathroom scale as he is rising in an elevator, if the bathroom scale reads 1500 N, what is the acceleration of the elevator?

1500 N



F_w

Given:

$$m = 150 \text{ kg}$$

$$g = 9.8 \text{ m/s}^2$$

$$F_{\text{net}} = 1500 \text{ N} - F_w$$

$$a = ?$$

Formula:

$$F_w = m \cdot g$$

$$F_{\text{Net}} = m \cdot a$$

Sample Problem (cont'd)

$$F_w = 150\text{kg} \cdot 9.8 \frac{\text{m}}{\text{s}^2}$$

$$F_w = 1470 \frac{\text{kg} \cdot \text{m}}{\text{s}^2}$$

$$1500\text{N} \cdot \frac{1 \frac{\text{kg} \cdot \text{m}}{\text{s}^2}}{1\text{N}} - 1470 \frac{\text{kg} \cdot \text{m}}{\text{s}^2} = 150\text{kg} \cdot a$$

$$30 \frac{\text{kg} \cdot \text{m}}{\text{s}^2} = 150\text{kg} \cdot a$$

$$a = \frac{30 \frac{\text{kg} \cdot \text{m}}{\text{s}^2}}{150\text{kg}}$$

Answer: 0.2 m/s²

Example 11

- An 80 kg man on ice skates pushes on a 40 kg boy, also on skates, with a force of 100 N. The force exerted by the boy on the man is
 - A. 200 N
 - B. 100 N
 - C. 50 N
 - D. 40 N
 - E. Zero unless the boy pushes back

Example 12

- A body of weight, W , rests on the surface of the earth. What force does the body exert on the earth?
 - A. W
 - B. Greater than W
 - C. Less than W
 - D. $9.81 W$
 - E. zero

Example 13

A **0.065-kg** tennis ball moving to the right with a speed of **15 m/s** is struck by a tennis racket, causing it to move to the left with a speed of **15 m/s**. If the ball remains in contact with the racquet for **0.020 s**, *what is the magnitude of the average force experienced by the ball?*

Sol: 97.5 N

Example 14

Q2- Mike is driving his **2500 kg BMW** down the street at **52 km/h**. He suddenly notices that there is a school zone ahead, so he hits the brakes to slow down to **24 km/h**. If he slowed down over a distance **145 m** for **3 seconds**. *Determine the average force applied by the brakes.*

Sol:-6483 N

Friction Force

- When a force attempts to slide a body along a surface, a frictional force is exerted on the body by the surface.
 - The frictional force is parallel to the surface and directed so as to oppose the sliding.
 - It is due to the bonding between the body and the surface
 - If the body does not slide, the frictional force is a static frictional force
 - If there is sliding, the frictional force is kinetic frictional force.

Static friction coefficient

- If the body is not moving, the static friction is equal in magnitude to the component of the applied force which is parallel to the surface but opposite in direction.

Static friction coefficient

- There is a maximum value for static friction which is given by the equation

$$F_{F_{s,\max}} = \mu F_N$$

- $F_{F_{s,\max}}$ = Maximum static frictional force
- μ = coefficient of static friction
- F_N = Normal force (amount of force pushing the surfaces into contact)

Example 15

- A bedroom bureau with a mass of 45 kg, including drawers and clothing, rests on the floor. If the coefficient of static friction between the bureau and the floor is 0.45, what is the minimum horizontal force a person must apply to start the bureau moving?

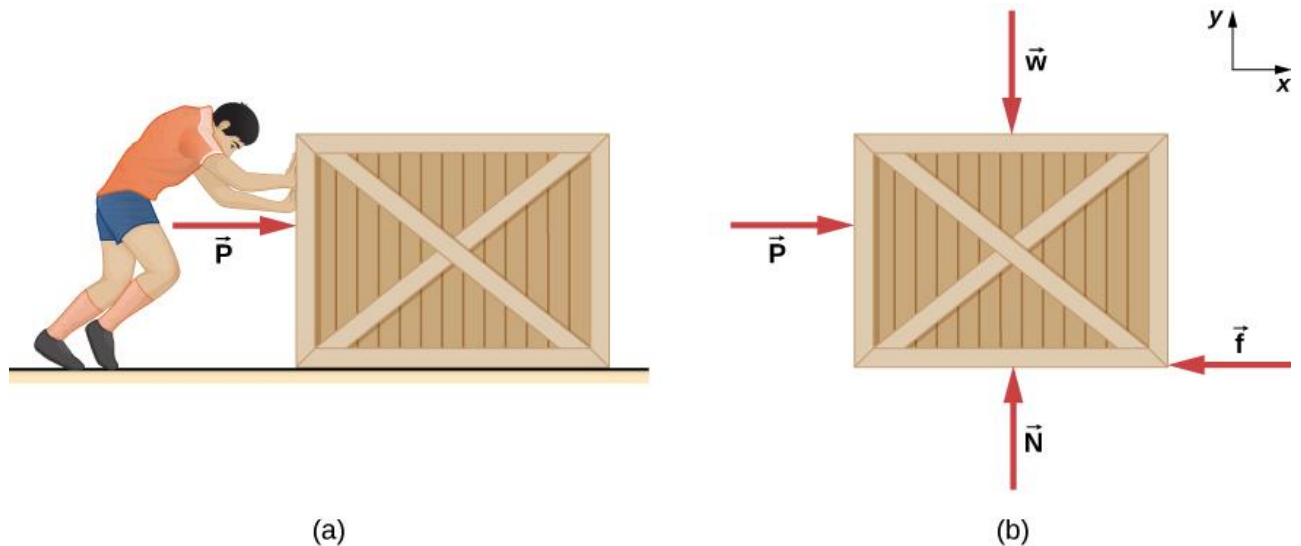


Friction

- If the component of the applied force that is parallel to the surface exceeds the maximum static frictional force, the object will begin to slide along the surface.
- When the object begins to slide along the surface, there is no longer static friction instead you have kinetic friction

Example 16

- A worker pushes horizontally on a 35 kg crate with a 150 N force. The coefficient of static friction between the crate and the floor is 0.37. What is the maximum static frictional force? Will the crate begin to move?



127 N

Yes

1. Which of Newton's Laws of Motion is called the Law of Inertia?

- A) 1st
- B) 3rd
- C) 2nd

2. Which of Newton's Laws of Motion states that objects at rest tend to stay at rest and objects in motion tend to stay in motion unless acted upon by an unbalanced force?

- A) 1st
- B) 2nd
- C) 3rd

3. The tendency of a moving object to continue in a straight line or a stationary object to remain in place is called -

- A) reaction
- B) action
- C) force
- D) Inertia

4. Which of Newton's Laws of Motion states that force equals mass times acceleration?

- A) 2nd
- B) 1st
- C) 3rd

5. According to Newton's 2nd Law of Motion, force equals -

- A) mass divided by acceleration
- B) mass plus acceleration
- C) mass times acceleration
- D) mass subtract acceleration

6. Which of Newton's Laws of Motion states that for every action, there is an equal and opposite reaction?

- A) 1st
- B) 3rd
- C) 2nd

7. Which of the following is defined as a push or a pull?

- A) deceleration
- B) acceleration
- C) mass
- D) Force

8. When a teacher stands at the front of the class, the force of gravity pulls her toward the ground. The ground pushes back with an equal and opposite force. This is an example of which of Newton's Laws of Motion?

- A) 3rd
- B) 1st
- C) 2nd

9. An object's inertia depends on its mass.

- A) True
- B) False

10. How much force is needed to accelerate a 90 kg skier 2 m/s²?

- A) 92 N
- B) 45 kg/ms²
- C) 180 N
- D) 180 kg/ms²