Financial Mathematics

Financial mathematics is the application of mathematical methods to financial problems.

1. **Simple interest (Definition)**

   In financial transactions, interest is an amount paid by a borrower to a lender or investor for the use of money over a period of time. Interest that is paid as a percent of the amount borrowed or invested is called simple interest. The formula for simple interest is the following:

   $$I = Prt$$

   Where I = interest earned (or Owed)
   P = Principal invested (or borrowed)
   r = annual interest rate (in decimal)
   t = time in years

   Example:

   Suppose $500 is invested for 2 years at 6 percent simple interest per year. Find the simple interest earned at the end of 2 years.

   Solution:

   $$I = 500(0.06)(2) = 60$$

   Hence, the simple interest earned at the end of 2 years is $60.

The Accumulated amount (Definition)

The accumulated amount A of an investment (or debt) is the principal plus interest. In the case of simple interest, the formula for the accumulated amount is as follows:

$$A = P + I = P + Prt = P(1 + rt)$$

Example:

Suppose $500 is invested for 2 years at 6 percent simple interest per year. Find the accumulated amount of the investment.

Solution:

$$A = 500(1 + 0.06(2)) = 560$$

Hence, the investment of $500 is worth $560 in 2 years.

Example:

A person borrows $2500 at 8% simple interest per year. Find the accumulated amount A of the debt in   a) 6 months;   b) 1.5 year

Solution:

a) $6\ months = \dfrac{6}{12} years = 0.5\ years$

So $A = 2500(1 + (0.08)(0.5) = \$2600$

b) $A = 2500(1 + 0.08(1.5)) = \$2800$

Present Value in Simple Interest: (Definition)

The principal P is also called the present value, and accumulated amount A is also called the future value. In the case of simple interest, the formula for the present value is

$$P = \frac{A}{1 + rt}$$

Example:

It is desired that the value of an investment at 5 percent annual simple interest should be $12,000 in 4 year. What amount must be invested now?

Solution:

$$P = \frac{12000}{1 + 0.05(4)} = 10000$$

Hence, an investment of $10,000 will yield 12,000 in 4 years.

H.W: A person borrows $1000 at 9.5 percent simple interest per year. Find the interest owed in   A) 10 months   B) 4.5 years

2. Compound Interest: Definition

Interest that is paid on both the principal and the accrued interest is called compound interest. In compound interest transactions, interest is computed over regular intervals.

Accumulated amount in compound interest (Definition)

Suppose a principal P is invested at an annual interest rate r, compounded k times per year. Then the accumulated amount A in t years is:

$$A = P\left(1 + \frac{r}{k}\right)^{kt}$$

P = Principal invested (or borrowed)
r = annual interest rate (in decimal)
t = time in years
k = number of compounds per year

Example:

Someone invests $2500 at an 8 percent annual rate compounded monthly. Find the accumulated amount in   A) 6 months   B) 10 years

Solution:

A)   6 months=0.5 years

   Compounded monthly...... k=12

$$A = 2500\left(1 + \frac{0.08}{12}\right)^{12(0.5)} = \$2601.68$$

B) $A = 2500\left(1 + \frac{0.08}{12}\right)^{12(10)} = ?$

Present Value in compound interest:

In compound interest, as in simple interest, the principal P and the accumulated amount A are also called the present value and future value, respectively. And the present value can be found by:

$$P = \frac{A}{\left(1+\frac{r}{k}\right)^{kt}} = A\left(1+\frac{r}{k}\right)^{-kt}$$

Example:

It is desired that the value of an investment at 5 percent annual rate, compounded monthly, should be $12,000 in 4 years. What amount must be invested now?

Solution:

$$P = \frac{12,000}{\left(1+\frac{0.05}{12}\right)^{12(4)}} = \$9828.85$$

Under 5 percent simple interest, the present value was $10,000. Hence, under compound interest, a smaller present value will yield the same future value.