Roots of Nonlinear Equations

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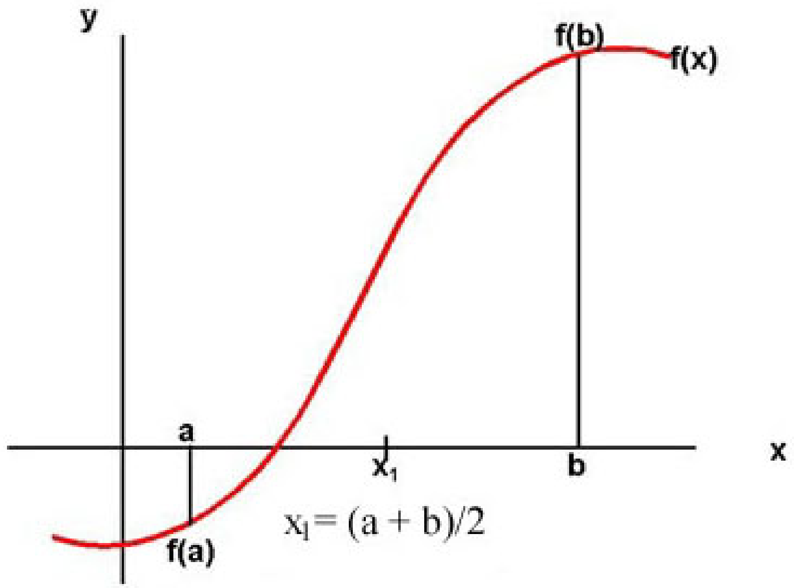
The root of nonlinear function is the value of x that makes the function $f(x) = 0$. It is also called zero value solution of nonlinear system. Many methods are available for computing the root of nonlinear system, the most common are:

1. Bisection Method
2. Newton-Raphson Method

Bisection Method

Bisection Method

Bisection Method is a numerical method for estimating the root of polynomial **f(x)**, assuming that **f(x)** is continuous. It repeatedly bisects an interval and selects a new subinterval (smaller interval) in which a root must lie for further processing. It is one of the simplest and most reliable but it is not the fastest method, however, its convergence is guaranteed. It is based on Intermediate Value Theorem.



Bisection Method Cont…

Bisection Method Algorithm to find the root of nonlinear equations, given a continuous function f(x). It consists of two parts:

The first part is the algorithm to find the interval [a,b] within which the root exists such that $a < b$ and $f(a) \times f(b) < 0$

The second part is the algorithm for implementing the Bisection Method to find the root.

Bisection Method Algorithm

Step 1: Start

Step 2: Define the function f(x), the interval [a,b] and the error ε, for example $f(x) = x^2 - 1.2, a = 1, b = 2 \; and \; \varepsilon = 0.00001$

Step 3: Set the initial value of the root as $x_{old} = \frac{a+b}{2}$

Step 4: For iteration $i = 1$

Step 5: Calculate $f(x_{old})$

Step 6: $If \; (f(x_{old}) = 0) \; Then$

$x_{new} = x\_old$

$x_{new} is \; an \; exact \; root$

$Print \; x\_new$

$goto \; step \; 10$

$Else \; If \; ((f(x_{old}) < 0) \; Then$

$a = x_{old}$

$Else \; If \; ((f(x_{old}) > 0) \; Then$

$b = x_{old}$

$End \; if$

Bisection Method Algorithm Cont…

Step 7: Calculate x_{new} $after\ the\ apdates\ of\ a\ and\ b$

$$x_{new} = \frac{a+b}{2}$$

Step 8: Calculate the error err $= |x_{new} - x_{old}|$

Step 9: $If\,(err \leq \varepsilon)\,Then$

$goto\ step\ 10$

$Else$

$i = i + 1$

$x_{old} = x\_new$

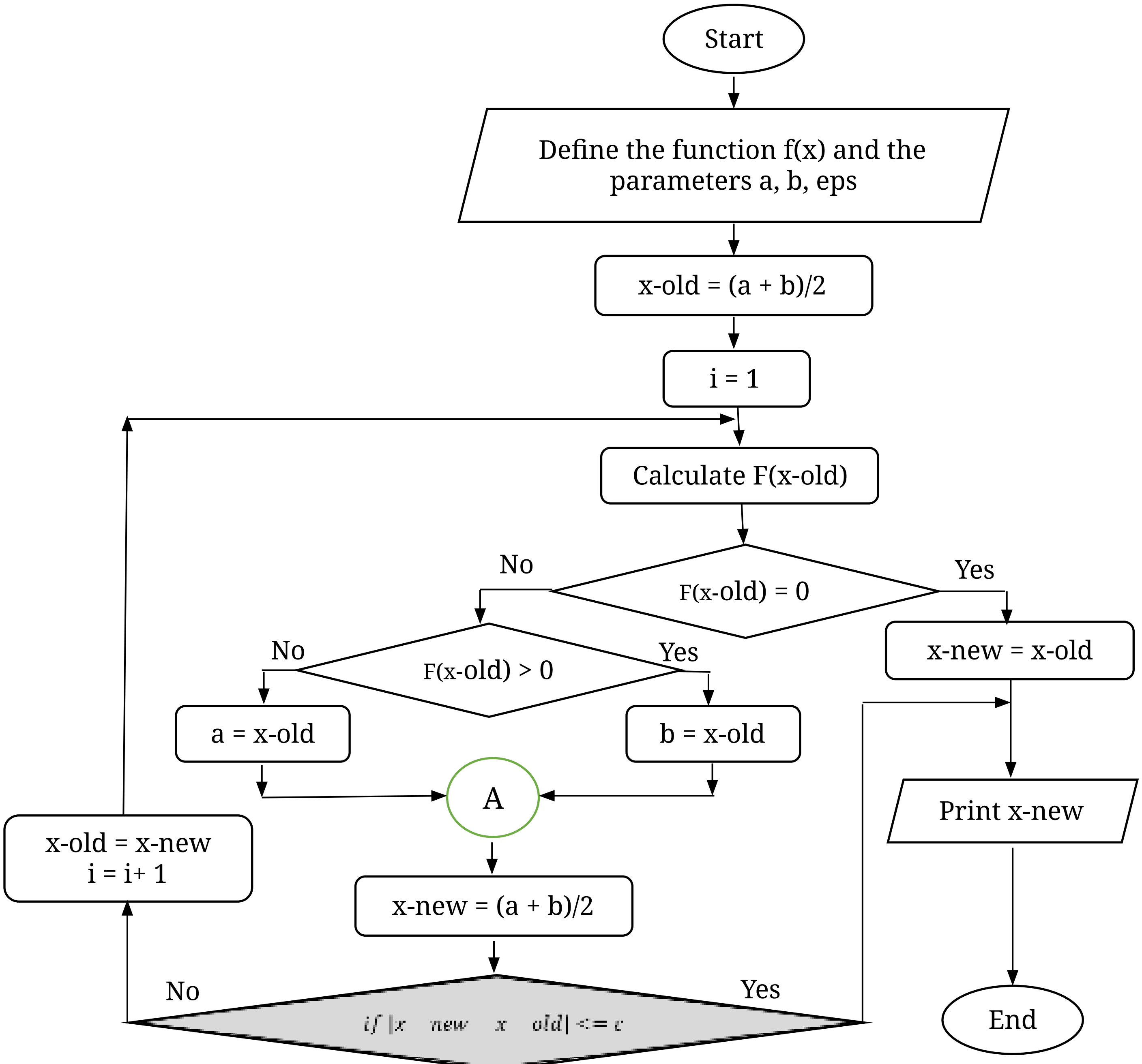
$goto\ step\ 5$

$End\ if$

Step 10: $Print\ x\_new$

Step 11: End

Bisection Method Flowchart



Example 2

Use bisection method to find the root of the following equation:

$f(x) = 3x + \sin x - e^x, a = 0.3, b = 0.4 \; and \; \varepsilon = 0.002,$

calculate sin(x) in radius.

$i = 1$

$x_1 = \frac{a+b}{2} = \frac{0.3+0.4}{2} = 0.35$

$f(x_1) = 3(0.35) + \sin(0.35) - e^{0.35}$= -0.026170

$a = 0.35 \; because \; f(x_1) < 0$

$i = 2$

$x_2 = \frac{a+b}{2} = \frac{0.35+0.4}{2} = 0.375$

$err = |x_2 - x_1| = |3.75 - 3.5| = 0.25$

Since $err > \varepsilon \; then \; repeat$

$f(x_2) = 3(0.375) + \sin(0.375) - e^{0.375}$= 0.036281

b= $0.375 \; because \; f(x_2) > 0$

$i = 3$

Repeat as shown in the following table.

EXAMPLE: Bisection Method

Consider $f(x) = 3x + sinx - e^{x}, a = 0.3, b = 0.4\ and\ \varepsilon = 0.002$

i	a	b			
1	0.3	0.4	0.35		- 0.026170
2	0.35	0.4	0.375	0.025	0.036281
3	0.35	0.375	0.3625	0.0125	0.005196
4	0.35	0.3625	0.35625	0.00625	-0.010452
5	0.35625	0.3625	0.359375	0.003125	- 0.002620
6	0.359375	0.3625	0.3609375	0.0015	0.001290

Since $|x_{i+1} - x_i| = 0.0015 < 0.002\ Then\ the\ root\ is\ 0.3609375$

Homework

Use Bisection Method to calculate the root of the following non-linear equations.

1. $f(x) = xlog(x) - 1.2, a = 1, b = 2 \; and \; \varepsilon = 0.004$
2. $f(x) = x^3 + 3x - 5, a = 1, b = 2 \; and \; \varepsilon = 0.002$
3. $f(x) = x^2 - 10x + 10, a = 1, b = 2 \; and \; \varepsilon = 0.004$