Integrations (Integrals)

derivative

$\frac{d}{dx}[x^2] = 2x$, $\frac{d}{dx}[x^2+\pi] = 2x$

$\frac{d}{dx}[x^2+1] = 2x$, $\frac{d}{dx}[x^2+c] = 2x$

Antiderivative

We are doing the oppsite of the derivative operator, What is $2x$ the derivative of?

x^2 , x^2+1 , $x^2+\pi$

x^2+c , This is the antiderivative

$\int 2x\,dx = \underbrace{x^2}_{F(x)}+c$ antiderivative of $2x$

$F(x)$ ↓ F is antiderivative of $f(x)$

Indefinite integral of $2x$

Indefinite integral

Def: A (differentiable) function F is called an antiderivative of f on an interval I if

$\boxed{F'(x) = f(x)}$, $\forall x \in I$

the antiderivative function is called indefinite integral & denoted by $F(x)$

$F(x)+c = \int f(x)\, dx \longrightarrow$ varible of integration

integral sign ; integrand

e.g Find the antiderivative of $f(x) = 2x$

antiderivative $F(x) = x^2$ $\Rightarrow F'(x) = 2x = f(x)$

$F'(x) = 2x = f(x)$

But $F(x) = x^2 + 1$

& $F(x) = x^2 - 1$, $F(x) = x^2 + 107$ are also antiderivative of $2x$ as $F'(x) = 2x$

There fore, we need to have general antiderivative of the given function

* If F is an antiderivative of f on an interval I, then the most general antiderivative of f is

$F(x) + c$, where c is arbitrary constant

that is $\int f(x)\,dx = F(x) + c$

$F'(x) = f(x)$

Properties of Indefinite Integrals

① $\int c\, f(x)\, dx = c \int f(x)\, dx$, c is a constant.

② $\int (f(x) \mp g(x))\, dx = \int f(x)\, dx \mp \int g(x)\, dx$

③ $\int (1)\, dx = x + c$ ④ $\int k\, dx = kx + c$, k, c (constant)

⑤ $\int x^{n}\, dx = \frac{1}{n+1} x^{n+1} + c$, $n \neq -1$

⑥ $\int \frac{1}{x}\, dx = \ln|x| + c$, ⑦ $\int x^{-1}\, dx = \ln|x| + c$

⑧ $\int x^{-n}\, dx = \frac{1}{-n+1} x^{-n+1} + c$, $n \neq 1$

⑨ $\int \frac{1}{ax+b}\, dx = \frac{1}{a} \ln|ax+b| + c$

⑩ $\int x^{p/q}\, dx = \frac{1}{\frac{p}{q}+1} x^{\frac{p}{q}+1} + c = \frac{q}{p+q} x^{\frac{p+q}{q}} + c$

Trig Functions

① $\int \cos x\, dx = \sin x + c$ ② $\int \sin x\, dx = -\cos x + c$

③ $\int \sec^2 x\, dx = \tan x + c$ ④ $\int \sec x \tan x\, dx = \sec x + c$

⑤ $\int \csc x \cot x\, dx = -\csc x + c$ ⑥ $\int \csc^2 x\, dx = -\cot x + c$

~~Some other examples Trig Integrals.~~

Inverse Trig. Functions

① $\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right)+c$

② $\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c$

③ $\int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right)+c$

The other 3 are similar

④ $\int a^x\,dx = \frac{a^x}{\ln a}+c$

⑤ $\int e^x\,dx = e^x+c$