examples ①

① $\int \frac{1}{2x-3}\,dx = \frac{2}{2}\int \frac{1}{2x-3}\,dx = \frac{1}{2}\int \frac{2}{2x-3}\,dx$

$= \frac{1}{2}\ln|2x-3| + C$

② $\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx = -\int \frac{-\sin x}{\cos x}\,dx$

$= -\ln|\cos x| + C$

③ $\int \sin^3 x \cos x\,dx = \int (\sin x)^3 \cos x\,dx$

$= \frac{\sin^4 x}{4} + C = \frac{1}{4}\sin^4 x + C$

④ $\int \frac{1}{4+x^2}\,dx = \frac{1}{2}\tan^{-1}\frac{x}{2} + C$

⑤ $\int \frac{1}{2+x^2}\,dx = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + C$

⑥ $\int \frac{e^{\frac{1}{x}}}{x^2}\,dx = -e^{\frac{1}{x}} + C$

$y = 2^{\bar{5}x+7}$ $\quad y' = 2^{\bar{5}x+7}\ln 2(\bar{5})$

⑦ $\int k^{ax+b}\,dx = \frac{1}{a\ln k} + C$ $\quad \int 2^{\bar{5}x+7}\,dx$

$= \frac{1}{\bar{5}\ln 2}\,2^{\bar{5}x+7} + C$

⑧ $\int \frac{1}{(\bar{5}x+4)^2}\,dx = \int (\bar{5}x+4)^{-2}\,dx = \frac{1}{\bar{5}}\frac{(\bar{5}x+4)^{-1}}{-1}$

$= -\frac{1}{\bar{5}}(\bar{5}x+4)^{-1}$

$= -\frac{1}{\bar{5}(\bar{5}x+4)}$

9) $\int \frac{1}{x\sqrt{x^2-4}}\,dx = \frac{1}{2}\sec^{-1}\left(\frac{x}{2}\right)+C \quad , x>2$

$$\int \frac{-dx}{\sqrt{1-x^2}} = \cos^{-1}x + C$$

$$\int \frac{-dx}{1+x^2} = \cot^{-1}x + C$$

$$\int \frac{-dx}{x\sqrt{x^2-1}} = \csc^{-1}x + C$$

10) $\int \frac{e^{2x}}{(e^{2x}+1)}\,dx = \int \frac{\frac{2}{2}e^{2x}}{(e^{2x}+1)}\,dx = \frac{1}{2}\ln|e^{2x}+1| + C$

11) $\int \sqrt{x}\,\csc^2(1+x^{\frac{3}{2}})\,dx = \frac{2}{3}\int\left(\frac{3}{2}x^{\frac{1}{2}}\right)\csc^2(1+x^{\frac{3}{2}})\,dx$

$$= -\frac{2}{3}\cot(1+x^{\frac{3}{2}}) + C$$

12) $\int \sec^2\left(\frac{2x}{3}\right)dx = \frac{3}{2}\int \frac{2}{3}\sec^2\left(\frac{2x}{3}\right)dx = \frac{3}{2}\tan\left(\frac{2x}{3}\right) + C$

13) $\int \csc(5x)\cot(5x)\,dx = -\frac{1}{5}\csc(5x) + C$ ✓

14) $\int \frac{1}{\sqrt{16-x^2}}\,dx = \sin^{-1}\left(\frac{x}{4}\right) + C$ $\left\{ \int \frac{1}{\sqrt{16\left(1-\frac{x^2}{16}\right)}}\,du \right.$

$$= \int \frac{1}{4\sqrt{1-\left(\frac{x}{4}\right)^2}}\,du$$

$$= \sin^{-1}\left(\frac{x}{4}\right)$$

15) $\int \frac{1}{3+x^2}\,dx = \frac{1}{\sqrt{3}}\cot^{-1}\left(\frac{x}{\sqrt{3}}\right) + C$

16 $\int \frac{\tan x}{\sin x \cos x}\, dx = \int \frac{\sin x}{\cos x} \cdot \frac{1}{\sin x \cos x}\, dx$

$\Rightarrow \int \frac{1}{\cos^2 x}\, dx = \int \sec^2 x\, dx = \tan x + c$

$\int \frac{\cos x}{\sin x}\, dx = \ln \sin x + c$

17 $\int (\ln x)^2 \frac{1}{x}\, dx = \frac{1}{3}(\ln x)^3 + c$

18 $\int e^{\tan x} \sec^2 x\, dx = e^{\tan x} + c$

19 $\int \sec^2(\sqrt{5}x - 3)(\sqrt{5})\, dx = \tan(\sqrt{5}x - 3) + c$

$\int \frac{1}{4 + x^2}\, dx = \int \frac{1}{4\left(1 + \frac{x^2}{4}\right)}\, dx$

$= \frac{1}{4} \int \frac{1}{1 + \left(\frac{x}{2}\right)^2}\, dx$

$= \frac{1}{2} \int \frac{\frac{1}{2}}{1 + \left(\frac{x}{2}\right)^2}\, dx = \frac{1}{2} \tan^{-1} \frac{x}{2}$

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$$\int \sqrt{\cot x}\ \csc^2 x\, dx = \frac{-(\cot x)^{\frac{3}{2}}}{\frac{3}{2}} + c$$

~~$\int \sin x \sec^2(\cos x)\, dx = \tan(\cos x) + c$~~

$$\int \sec^3 x \tan x\, dx = \int \sec^2 x \sec x \tan x\, dx$$

$$= \frac{\sec^3 x}{3} + c$$

$$\int \frac{\cos\sqrt{x}}{\sqrt{x}}\, dx = \int \frac{2}{2} \frac{\cos\sqrt{x}}{\sqrt{x}}\, dx$$

$$= 2\sin\sqrt{x} + c$$

$$\int \sin x \sec^2(\cos x)\, dx = -\tan(\cos(x)) + c$$