The another random variables is

Continuous random variables.

The special prob. dist. for Continuous Random variables are

1- The uniform density:

Def:- a random variable X has a uniform density and it is referred to as a continuous uniform random variable iff its prob. density is given by:

$$f(x;\alpha,\beta)=\begin{cases}\frac{1}{\beta-\alpha} & \text{for } \alpha\le x\le\beta\\ 0 & o.w.\end{cases}$$

Theorem: The mean and variance of this dist. is given by $\mu=\frac{\alpha+\beta}{2}$ and variance $(\sigma_x^2)=\frac{(\beta-\alpha)^2}{12}$

i.e. $X\sim U(\alpha,\beta)$ it is parameters or Constant.

for Ex: $X\sim U(2,4)$ Find the ① uniform dist.
② The mean of this dist.
③ The variance " " "

Sol: $f(x;\alpha,\beta)=\begin{cases}\frac{1}{\beta-\alpha} & \alpha<x<\beta\\ 0 & o.w.\end{cases}$

① $f(x;2,4)=\begin{cases}\frac{1}{2} & 2<x<4\\ 0 & o.w.\end{cases}$

② $\mu=\frac{\alpha+\beta}{2}=\frac{2+4}{2}=\frac{6}{2}=3$

③ variance$(x)=var(x)=\frac{(\beta-\alpha)^2}{12}=\frac{(4-2)^2}{12}=\frac{2^2}{12}=\frac{4}{12}=\frac{1}{3}$

2- The Gamma distribution

Def:- A random variable X has a gamma dist. and it is referred to as a gamma random variable iff its prob. dist. is given by

$$f(x) = \begin{cases} \frac{1}{\beta^{\alpha}\Gamma\alpha} \cdot x^{\alpha-1} e^{-x/\beta} & \text{for } x>0 \\ & \alpha,\beta>0 \\ 0 & \text{o.w.} \end{cases}$$

$\Gamma\alpha$ is equal to $(\alpha-1)!$ for ex. $\Gamma 3 = (3-1)! = 2! = 2$

The mean of this dist. is $\mu = E(X) = \alpha\beta$

$var(x) = \alpha\beta^2$

Then $X \sim G(\alpha,\beta) \Rightarrow$ are parameters.

Ex: $X \sim G(3,5)$, Find the dist.

& mean

& $var(X)$

Sol^n

$$f(x;\alpha,\beta) = \begin{cases} \frac{1}{\beta^{\alpha}\Gamma\alpha} x^{\alpha-1} e^{-\frac{x}{\beta}} & x>0 \\ & \alpha,\beta>0 \\ 0 & \text{o-w} \end{cases}$$

$$f(x;3,5) = \begin{cases} \frac{1}{5^3\Gamma 3} x^{3-1} e^{-\frac{x}{5}} \\ 0 & \text{o-w.} \end{cases} = \begin{cases} \frac{1}{5^3(2)} x^{2} e^{-\frac{x}{5}} & x>0 \\ 0 & \text{o.w.} \end{cases}$$

$E(x) = \alpha\beta = 3(5) = 15$

$var(x) = \alpha\beta^2 = 3(5^2) = 3(25) = 75$

3- Exponential distribution :-

Def:- A random variable X has an exponential dist. iff prob. is given by $f(x;\theta) = \begin{cases} \frac{1}{\theta} e^{-\frac{x}{\theta}} & \text{for } x > 0 \\ 0 & \text{o.w.} \end{cases}$

or we can write another formula

$$f(x;\beta) = \begin{cases} \frac{1}{\beta} e^{-\frac{x}{\beta}} & \text{for } x > 0 \\ 0 & \text{o.w.} \end{cases}$$

we can ~~d~~erivative this dist. from the gamma dist.

where $\alpha = 1$

$$f(x;\alpha,\beta) = \begin{cases} \frac{1}{\beta^{\alpha}\Gamma\alpha} x^{\alpha-1} e^{-\frac{x}{\beta}} \\ 0 & \text{o.w.} \end{cases} \Leftarrow \text{Gamma dist.}$$

$$f(x;1,\beta) = \begin{cases} \frac{1}{\beta^{1}\Gamma 1} x^{1-1} e^{-\frac{x}{\beta}} \\ 0 & \text{o.w} \end{cases} = \begin{cases} \frac{1}{\beta} e^{-\frac{x}{\beta}} & x > 0 \\ 0 & \text{o.w.} \end{cases}$$

Because $\Gamma 1 = (1-1)! = 0! = 1$, $x^{0} = 1$

The Mean $= \mu = E(x) = \beta$

$var(x) = \beta^2 = \sigma^2$

Ex:- $X \sim Exp(5)$, find the dist. & mean & variance

$$1- f(x;\theta) = \begin{cases} \frac{1}{\theta} e^{-\frac{x}{\theta}} & x > 0 \\ 0 & \text{o.w.} \end{cases}$$

$$= \begin{cases} \frac{1}{5} e^{-\frac{x}{5}} & x > 0 \\ 0 & \text{o.w} \end{cases}$$

$Mean = E(x) = \theta = 5$

$var(x) = \sigma^2 = \theta^2 = (5)^2 = 25$

4- Chi-square distribution χ^2

Def: A random variable X has a chi-square dist. iff its prob. dist. is given by:

$$f(x) = \begin{cases} \dfrac{1}{2^{\frac{v}{2}} \Gamma\left(\frac{v}{2}\right)} x^{\frac{v}{2}-1} e^{-\frac{x}{2}} & \text{for } x > 0 \\ 0 & o.w. \end{cases}$$

The mean $= E(x) = \mu = v$ where v means the degree of freedom

$var(x) = \sigma_x^2 = 2v$

$$X \sim \chi^2(v)$$

Ex: $X \sim \chi^2(4)$, find the dist., mean(x), varian(x)

$$f(x) = \begin{cases} \dfrac{1}{2^{2} \Gamma(2)} x^{2-1} e^{-\frac{x}{2}} & x > 0 \\ 0 & o.w. \end{cases}$$

$$= \begin{cases} \dfrac{1}{4} x e^{-\frac{x}{2}} & x > 0 \\ 0 & o.w. \end{cases}$$

The Mean $= \mu = v = 4$

$var(x) = \sigma^2 x = 2(4) = 8$

5- Beta distribution

Def: A random variable x has a beta distribution iff its prob. distr is given by

$$f(x) = \begin{cases} \dfrac{\Gamma(\alpha+\beta)}{\Gamma\alpha\,\Gamma\beta}\, x^{\alpha-1}(1-x)^{1-\beta} & 0<x<1 \\ 0 & \text{o.w} \end{cases}$$

$$\mu = E(x) = \text{Mean} = \frac{\alpha}{\alpha+\beta}$$

$$\text{var}(x) = \sigma_x^2 = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

Ex: $X \sim \text{Beta}(\alpha,\beta) \Rightarrow X \sim \text{Beta}(3,5)$

$$f(x) = \begin{cases} \dfrac{\Gamma 8}{\Gamma 3\,\Gamma 5}\, x^{2}(1-x)^{4} & 0<x<1 \\ 0 & \text{o.w} \end{cases}$$

$$\mu = E(x) = \frac{\alpha}{\alpha+\beta} = \frac{3}{8}$$

$$\text{var}(x) = \sigma_x^2 = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} = \frac{15}{8^2(9)=576} = \frac{15}{576}$$

$$= 0.026$$

6- Normal distribution

Def: A random variable x has a normal distribution iff its prob. dist. is given by

$$f(x; \mu, \sigma) = \begin{cases} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} & -\infty \le x \le \infty \\ & \sigma > 0 \\ 0 & \text{o.w.} \end{cases}$$

The Mean $= E(x) = \mu = \mu$

$var(x) = \sigma_x^2 = \sigma^2$

Ex: $x \sim N(3,3)$, find the dist.

Mean (x)

variance (x)

$$f(x, 3, 3) = \begin{cases} \frac{1}{3\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-3}{3}\right)^2} & -\infty \le x \le \infty \\ 0 & \text{o.w.} \end{cases}$$

Mean $= \mu = 3$

$var(x) = \sigma_x^2 = \sigma^2 = 3^2 = 9$

Note: if the random dist. $\mu = 0$ and $\sigma = 1$ is referred to as the standard normal dist.

Ex: $x \sim N(\mu, \sigma^2) \Rightarrow x \sim N(0,1)$ then

$$f(x, 0, 1) = \begin{cases} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} & -\infty \le x < \infty \\ 0 & \text{o.w.} \end{cases}$$