The measure of Central Tendency

- Grouped data

1- $\bar{x} = \frac{\sum x_i f_i}{\sum f_i}$

2- $\bar{G} = \sqrt[\sum f_i]{(x_1^{f_1})(x_2^{f_2}) \cdots (x_n^{f_n})}$

3- $\bar{H} = \frac{\sum f_i}{\frac{f_1}{x_1} + \frac{f_2}{x_2} + \cdots + \frac{f_n}{x_n}}$

Example

class	f_i	x_i	$f_i x_i$
60-62	5	61	305
63-65	18	64	1152
66-68	42	67	2814
69-71	27	70	1890
72-74	8	73	584
	$\sum f_i = 100$		$\sum f_i x_i = 6745$

1- $\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{6745}{100} = 67.45$

2- $\bar{G} = \sqrt[100]{(61)^5 (64)^{18} \cdots (73)^8} =$ H-w. 1

3- $\bar{H} = \frac{100}{\frac{5}{61} + \frac{18}{64} + \cdots + \frac{8}{73}} =$ H-w. 2

Theorems for the Mean

1- $\sum (y_i - \bar{y}) = 0$

sol: $\sum y_i - \sum \bar{y} = \sum y_i - n\bar{y} = \sum y_i - \cancel{n}\left(\frac{\sum y_i}{\cancel{n}}\right)$

$= \sum y_i - \sum y_i = 0$

H.w. $\sum f_i (y_i - \bar{y}) = 0 \Rightarrow \sum f_i y_i - \sum f_i \bar{y} = f_i \sum y_i - f_i n\bar{y}$

$= f_i n\bar{y} - f_i n\bar{y}$

$= 0$

2- If $z = ky$ then $\bar{z} = k\bar{y}$

sol: $z = ky \Rightarrow \sum z = \sum ky \Rightarrow \sum z = k\sum y$ (Both sides divided by n)

$\frac{\sum z}{n} = k\frac{\sum y}{n} \Rightarrow \bar{z} = k\bar{y}$

3- If $x_i = y_i + k$ then $\bar{x} = \bar{y} + k$

sol: $x_i = y_i + k \Rightarrow (\sum x_i = \sum y_i + \sum k) / n$

$\frac{\sum x_i}{n} = \frac{\sum y_i}{n} + \frac{\cancel{n}k}{\cancel{n}} \Rightarrow \bar{x} = \bar{y} + k$

4- If $z_i = x_i + y_i$ then $\bar{z} = \bar{y} + \bar{x}$

sol: $z_i = x_i + y_i \Rightarrow (\sum z_i = \sum x_i + \sum y_i) / n$

$\frac{\sum z_i}{n} = \frac{\sum x_i}{n} + \frac{\sum y_i}{n}$

$= \bar{x} + \bar{y} = \bar{y} + \bar{x}$

Median for grouped data

$$\tilde{M}e = L_i + \left(\frac{\frac{\sum f_i}{2} - f_i}{f_i \text{ median}}\right) * w$$

L_i = The Lower class boundary of the class number

$\sum f_i$ = Summation of frequency

f_i = Less than Cummulative distribution in the median class

w = class Width

f_i median = freq. of median

Example: Find the median from the freq. table below

classes	f_i	Less than	f_i
60 – 62	5	Less than 60	0
63 – 65	18	" " 63	5
66 – 68	42	" " 66	23
69 – 71	27	" " 69	65
72 – 74	8	" " 72	92
75 –		" " 75	100

$65 - 23 = 42$

$\sum f_i = 100$

$$\tilde{M}e = L_i + \left(\frac{\frac{\sum f_i}{2} - f_i}{f_i \text{ median}}\right) * w = 65.5 + \left(\frac{50 - 23}{42}\right) * 3$$

$$= 67.48$$

$L_i = 66 - 0.5 = 65.5$

$f_i = 23$

$\frac{\sum f_i}{2} = \frac{100}{2} = 50$

f_i median $= 65 - 23 = 42$

$w_i = 3$

Mode for grouped data

$$\bar{Mo} = L_i + \left(\frac{\Delta_1}{\Delta_1 + \Delta_2}\right) * w_i$$

$L_i \Rightarrow$ The lower class boundary of the class number.

$\Delta_1 \Rightarrow$ The freq. of the group befor the model class.

$\Delta_2 \Rightarrow$ 〃 〃 〃 〃 〃 after 〃 〃 〃.

$w_i \Rightarrow$ class width

Example: Find the mode from the freq. table

classes	f_i
31 – 40	1
41 – 50	2
51 – 60	5
61 – 70	15
71 – 80	25 ←
81 – 90	20
91 – 100	12

$\sum f_i = 80$

$$\bar{Mo} = L_i + \left(\frac{\Delta_1}{\Delta_1 + \Delta_2}\right) * w = 70.5 + \left(\frac{10}{10+5}\right) * 10$$

$$= 77.10$$

$L_i = 71 - 0.5 = 70.5$

$\Delta_1 = 25 - 15 = 10$

$\Delta_2 = 25 - 20 = 5$

$\sum f_i = 80$

$w = 10$

H.w.

① Find the mean, median and mode from this table.

Length(mm)	Frequency
150 − 154	5
155 − 159	2
160 − 164	6
165 − 169	8
170 − 174	9
175 − 179	11
180 − 184	6
185 − 189	3

② Find the mean for 2, 4, 6, 8, 10

" " median for 5, 5, 5, 6, 6, 7, 7, 8, 8, 9, 10, 10, 10, 4, 11, 12, 12, 13, 6, 7

" " mode for 24, 15, 18, 20, 18, 22, 24, 26, 18, 26, 24

Measures of Dispersion or variation

1- Rang = $X_{max} - X_{min}$

Ex: 9, 3, 2, 1, 10, 3, 8

$R = 10 - 1 = 9$

2- Variance (S^2): The variance of a set of observation $x_1, x_2, \ldots, x_n$ denoted by S^2 then

$$S^2 = \frac{\sum(x_i - \bar{x})^2}{n-1} = \frac{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}{n-1}$$

3- Standard deviation (SD): The SD of a set of observation $x_1, x_2, \ldots, x_n$ denoted by (S.)

$$S = \sqrt{S^2}$$

* If you want to prove $\sum(x_i-\bar{x})^2 = \sum x_i^2 - \frac{(\sum x_i)^2}{n}$

$$\sum(x_i-\bar{x})^2 = \sum(x_i^2 - 2x_i\bar{x} + \bar{x}^2)$$

$$= \sum x_i^2 - 2\bar{x}\sum x_i + n\bar{x}^2$$

$\bar{x} = \frac{\sum x_i}{n}$

$$= \sum x_i^2 - 2\frac{\sum x_i}{n}\cdot\sum x_i + n\frac{(\sum x_i)^2}{n^2}$$

$\sum x_i = n\bar{x}$

$$= \sum x_i^2 - 2\frac{(\sum x_i)^2}{n} + \frac{(\sum x_i)^2}{n}$$

$$= \sum x_i^2 - \frac{(\sum x_i)^2}{n}$$

4- Sum of square (SS)

$$SS = \sum(x_i-\bar{x})^2$$

Then $S^2 = \frac{SS}{n-1} = \frac{\sum(x_i-\bar{x})^2}{n-1}$

* The important properties of variance and standard deviation are

1/ if $x_i = y_i + k$ then $S^2x = S^2y$, $Sx = Sy$

2/ if $x_i = k y_i$ then $S^2x = k^2 S^2y$, $Sx = kSy$

To prove if $x_i = k + cy_i$ (c, k) are constant

$$S^2x = c^2 S^2y$$

$x_i = k + cy_i \Rightarrow \frac{\sum x_i}{n} = \frac{nk}{n} + c\frac{\sum y_i}{n}$

$$\bar{x} = k + c\bar{y}$$

$x_i = k + cy_i$

$x_i - \bar{x} = (k + cy_i) - \bar{x}$ subtract $\bar{x}$ from two sides

$\because \bar{x} = k + c\bar{y}$ Then

$$x_i - \bar{x} = (k + cy_i) - (k + c\bar{y}) = k + cy_i - k - c\bar{y}$$

$$x_i - \bar{x} = cy_i - c\bar{y} = c(y_i - \bar{y})$$

take Σ and square both sides

$$\Sigma(x_i-\bar{x})^2 = c^2\,\Sigma(y_i-\bar{y})^2$$ divid by sides by $n-1$

$$\frac{\Sigma(x_i-\bar{x})^2}{n-1} = c^2\,\frac{\Sigma(y_i-\bar{y})^2}{n-1}$$

$$\therefore\ S^2x = c^2\,S^2y$$

3/ if x, y are two independent variables and the variable Z is equal to summation them

i.e. if $Z = x_i + y_i$ then $S^2z = S^2x + S^2y$

4/ if two set of values that content from n_1, n_2 observation and have two variance S_1^2, S_2^2 respectively then $S^2p = \dfrac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}$ is called weight variance $S_1^2 = \dfrac{\Sigma(x_i-\bar{x})^2}{n-1} \Rightarrow (n-1)S_1^2 = \Sigma(x_i-\bar{x})^2$

or $S^2p = \dfrac{SS_1 + SS_2}{n_1+n_2-2}$

EX: Find the standard deviation from ungrouped data if $x_i = 9, 6, 8, 5, 7$ then $S^2 = \dfrac{\Sigma(x_i-\bar{x})^2}{n-1}$ and $S = \sqrt{s^2}$

x_i	$x_i-\bar{x}$	$(x_i-\bar{x})^2$
9	$9-7=2$	4
6	$6-7=-1$	1
8	$8-7=1$	1
5	$5-7=-2$	4
7	$7-7=0$	0
$\Sigma x_i=35$	$\Sigma(x_i-\bar{x})=0$	$\Sigma(x_i-\bar{x})^2=10$

$\bar{x} = \dfrac{\Sigma x_i}{n} = \dfrac{35}{5} = 7$

$S^2 = \dfrac{10}{4} = 2.5$

$S = \sqrt{s^2} = \sqrt{2.5} = \boxed{1.58}$

5/ Mean Deviation (M.D.)

ungrouped data

$$M.D. = \frac{\sum |x_i - \bar{x}|}{n}$$

grouped data

$$\frac{\sum f_i |x_i - \bar{x}|}{\sum f_i}$$

Ex1 for ungrouped data

Let a set of no. of observation are 2, 3, 6, 8, 11 find M.D.

Sol:

x_i	$x_i - \bar{x}$	$\lvert x_i - \bar{x} \rvert$
2	$2-6=-4$	$\lvert -4 \rvert = 4$
3	$3-6=-3$	$\lvert -3 \rvert = 3$
6	$6-6=0$	$\lvert 0 \rvert = 0$
8	$8-6=2$	$\lvert 2 \rvert = 2$
11	$11-6=5$	$\lvert 5 \rvert = 5$
$\sum x_i = 30$	$\sum (x_i - \bar{x}) = 0$	$\sum \lvert x_i - \bar{x} \rvert = 14$

$$\bar{x} = \frac{\sum x_i}{n} = \frac{30}{5} = 6$$

$$M.D. = \frac{\sum |x_i - \bar{x}|}{n} = \frac{14}{5} = \boxed{2.8}$$

Ex2 for grouped data

classes	f_i	x_i	$f_i x_i$	$x_i - \bar{x}$	$\lvert x_i - \bar{x} \rvert$	$f_i \lvert x_i - \bar{x} \rvert$
60-62	5	61	305	$61-67.4=-5.4$	5.4	27
63-65	18	64	1152	$64-67.4=-2.4$	2.4	43.2
66-68	42	67	2814	$67-67.4=-0.6$	0.6	25.2
69-71	27	70	1890	$70-67.4=3.6$	3.6	97.2
72-74	8	73	584	$73-67.4=6.6$	6.6	52.8
	$\sum f_i = 100$		$\sum f_i x_i = 6745$			245.4

$$\bar{x} = \frac{\sum x_i f_i}{\sum f_i} = \frac{6745}{100} = 67.4$$

$$M.D. = \frac{\sum f_i |x_i - \bar{x}|}{\sum f_i} = \frac{245.4}{100} = 2.45$$

6- standarized scores (Z_i)

$$Z_i = \frac{x_i - \bar{x}}{S}$$

7- Coefficient of variance (C.V.).

$$C.V. = \frac{S}{\bar{x}} \times 100$$

8- standard deviation of Mean $S\bar{x}$

$$S\bar{x} = \frac{S}{\sqrt{n}} = \sqrt{\frac{S^2}{n}}$$

Exr: Assume that the results for the fourth students examination of computer and statistics Find the C.V., Z_i, $S\bar{x}$

Computer	Statistics
15	17
20	24
23	19
10	30

Sol: we apply only for Computer and H.W. for statistics

x_i	$(x_i-\bar{x})^2$	$Z_i = x_i-\bar{x}/S$
15	15-17 = 4	15-17/5.7 = -0.35
20	20-17 = 9	
23	23-17 = 36	
10	10-17 = 49	H.W.
$\Sigma x_i = 68$	$\Sigma(x_i-\bar{x})^2 = 98$	

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{68}{4} = 17$$

$$S^2 = \frac{\Sigma(x_i-\bar{x})^2}{n-1} = \frac{98}{3} = 32.667$$

$$S = \sqrt{S^2} = \sqrt{32.667} = 5.7$$

a- $C.V. = \frac{S}{\bar{x}} * 100 = \frac{5.7}{17} * 100 = 33.53$

b- $Z_i = \frac{x_i - \bar{x}}{S} \longrightarrow$ H.W.

c- $S\bar{x} = \frac{S}{\sqrt{n}} = \frac{5.7}{\sqrt{4}} = \frac{5.7}{2} = 2.85$