Σ - Notation

The symbol $\sum_{i=1}^{n} x_i$ is used to sum of all the x_i from $i=1$ to n by definition $\sum_{i=1}^{n} x_i = x_1 + x_2 + \cdots + x_n$

1- $\sum_{i=1}^{n} x_i^2 = x_1^2 + x_2^2 + \cdots + x_n^2 \Rightarrow \sum_{i=1}^{n} x_i^2 \neq \left(\sum_{i=1}^{n} x_i\right)^2$

2- $\sum_{i=1}^{n} c\,x_i = cx_1 + cx_2 + \cdots + cx_n$ or $c\sum_{i=1}^{n} x_i \Rightarrow \sum_{i=1}^{n} cx = c\sum_{i=1}^{n} x_i$

3- $\sum_{i=1}^{n} c = c_1 + c_2 + \cdots + c_n = nc$

Let we have two variables X and Y then

1- $\sum_{i=1}^{n} x_i y_i \neq \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i$

2- $\sum_{i=1}^{n} \frac{x_i}{y_i} \neq \frac{\sum_{i=1}^{n} x_i}{\sum_{i=1}^{n} y_i}$

3- $\sum_{i=1}^{n} (x_i + y_i) = \sum_{i=1}^{n} x_i + \sum_{i=1}^{n} y_i$

4- $\sum \log x_i = \log x_1 + \log x_2 + \cdots + \log x_n$

Π - Notation

The symbol $\prod_{i=1}^{n} x_i$ is used to denote the product of all the x_i from $i=1$ to n by definition

$\prod_{i=1}^{n} x_i = x_1 * x_2 * \cdots * x_n$

1- $\prod_{i=1}^{n} a = a_1 * a_2 * \cdots * a_n = a^n$

2- $\prod_{i=1}^{n} a x_i = a^n \prod_{i=1}^{n} x_i$

3- $\prod_{i=1}^{n} x_i y_i = \prod_{i=1}^{n} x_i \prod_{i=1}^{n} y_i$

4- $\log \prod_{i=1}^{n} x_i = \sum_{i=1}^{n} \log x_i$

5- $\prod_{i=1}^{n} \sqrt{x_i} = \sqrt{\prod_{i=1}^{n} x_i}$

Example:① Given $x_1 = 1;\ x_2 = 3;\ x_3 = 5;\ x_4 = 7;\ x_5 = 9$

$y_1 = 1;\ y_2 = 5;\ y_3 = 10;\ y_4 = 3;\ y_5 = 2$

Find a- $\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} y_i$ b- $\sum_{i=1}^{n} x_i y_i$ c- $\sum_{i=1}^{n} (x_i + y_i)$

Sol:- ① $\sum_{i=1}^{5} x_i = x_1 + x_2 + \dots + x_5 = 1+3+5+7+9 = 25$

$\sum_{i=1}^{5} y_i = y_1 + y_2 + \dots + y_5 = 1+5+10+3+2 = 21$

$\therefore \sum x_i + \sum y_i = 25 + 21 = 46$

② $\sum x_i y_i = \sum_{i=1}^{5} (x_1 y_1 + x_2 y_2 + x_3 y_3 + \dots + x_5 y_5) = 105$

③ $\sum_{i=1}^{5} (x_i + y_i) = (x_1 + y_1) + (x_2 + y_2) + \dots + (x_5 + y_5) = 46$

Example: (2) Solve the problem:- ① $\sum_{i=1}^{n} (x_i - \bar{x})^2$, ② $\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})$

1- $\sum_{i=1}^{n} (x_i - \bar{x})^2 = \sum_{i=1}^{n} (x_i^2 - 2x_i\bar{x} + \bar{x}^2)$

$= \sum_{i=1}^{n} x_i^2 - 2\bar{x}\sum_{i=1}^{n} x_i + \sum \bar{x}^2$

$= \sum x_i^2 - 2\bar{x}\, n\bar{x} + n\bar{x}^2$

$= \sum_{i=1}^{n} x_i^2 - 2n\bar{x}^2 + n\bar{x}^2 = \sum_{i=1}^{n} x_i^2 - n\bar{x}^2$

$\cancel{\bar{x} = \frac{\sum x_i}{n}}$

$\sum x_i = n\bar{x}$

2- $\sum_{i=1}^{n}(x_i-\bar{x})(y_i-\bar{y}) = \sum(x_iy_i - x_i\bar{y} - y_i\bar{x} + \bar{x}\bar{y})$

$\bar{y} = \frac{\sum y_i}{n}$

$\sum y_i = n\bar{y}$

$= \sum_{i=1}^{n} x_iy_i - \bar{y}\sum_{i=1}^{n} x_i - \bar{x}\sum_{i=1}^{n} y_i + n\bar{x}\bar{y}$

$\bar{x} = \frac{\sum x_i}{n}$

$\sum x_i = n\bar{x}$

$= \sum_{i=1}^{n} x_iy_i - \bar{y}n\bar{x} - \cancel{\bar{x}n\bar{y}} + \cancel{n\bar{x}\bar{y}} = \sum x_iy_i - n\bar{x}\bar{y}$

Solve problem

1- write the terms in each of the following indicated sum.

a- $\sum_{j=1}^{6} x_j$; b- $\sum_{j=1}^{4}(y_i-3)^2$; c- $\sum_{j=1}^{n} a$; d- $\sum_{k=1}^{5} f_k x_k$ e- $\sum_{i=1}^{3}(x_i-a)$

a- $\sum_{j=1}^{6} x_j = x_1+x_2+x_3+x_4+x_5+x_6$

b- $\sum_{i=1}^{4}(y_i-3)^2 = (y_1-3)^2+(y_2-3)^2+(y_3-3)^2+(y_4-3)^2$

c- $\sum_{j=1}^{n} a = a_1+a_2+ - - - +a_n = na$

d- $\sum_{k=1}^{5} f_k x_k = f_1x_1 + f_2x_2 + - - - + f_5x_5$

e- $\sum_{i=1}^{3}(x_i-a) = (x_1-a)+(x_2-a)+(x_3-a)$

$\sum_{j=1}^{3} a = 3a$

Example: Express each of the following using the summation notation

a- $x_1^2 + x_2^2 + \cdots + x_{10}^2 = \sum_{i=1}^{10} x_i^2$

b- $(x_1 + x_1) + (x_2 + x_2) + \cdots + (x_8 + x_8) = \sum_{i=1}^{8} (x_i + x_i)$

c- $f_1 x_1^3 + f_2 x_2^3 + \cdots + f_{20} x_{20}^3 = \sum_{i=1}^{20} f_i x_i^3$

d- $a_1 b_1 + a_2 b_2 + \cdots + a_n b_n = \sum_{i=1}^{n} a_i b_i$

e- $f_1 x_1 y_1 + f_2 x_2 y_2 + \cdots + f_4 x_4 y_4 = \sum_{i=1}^{4} f_i x_i y_i$

H.W.

Q1: Two variables x and y assume the values $x_1 = 2;\ x_2 = 5;\ x_3 = 4;\ x_4 = 8$

$y_1 = -3;\ y_2 = -8;\ y_3 = 10;\ y_4 = 6$

Calculate: (a) $\sum_{i=1}^{4} x_i$ (b) $\sum_{i=1}^{4} y_i$ (c) $\sum_{i=1}^{4} x_i y_i$ (d) $\sum_{i=1}^{n} (x_i + y_i)(x_i - y_i)$

(e) $\sum_{i=1}^{4} x_i^2$ (f) $\sum_{i=1}^{4} y_i^2$ (g) $\sum_{i=1}^{4} x_i^2 - \sum y_i^2$

Q2: if $\sum_{i=1}^{6} x_i = -4$ and $\sum_{i=1}^{6} x_i^2 = 10$

calculate: (1) $\sum_{i=1}^{6} (2x_i - 3)$ (2) $\sum_{i=1}^{6} x_i (x_i - 1)$

(3) $\sum_{i=1}^{6} (x_i - 5)$