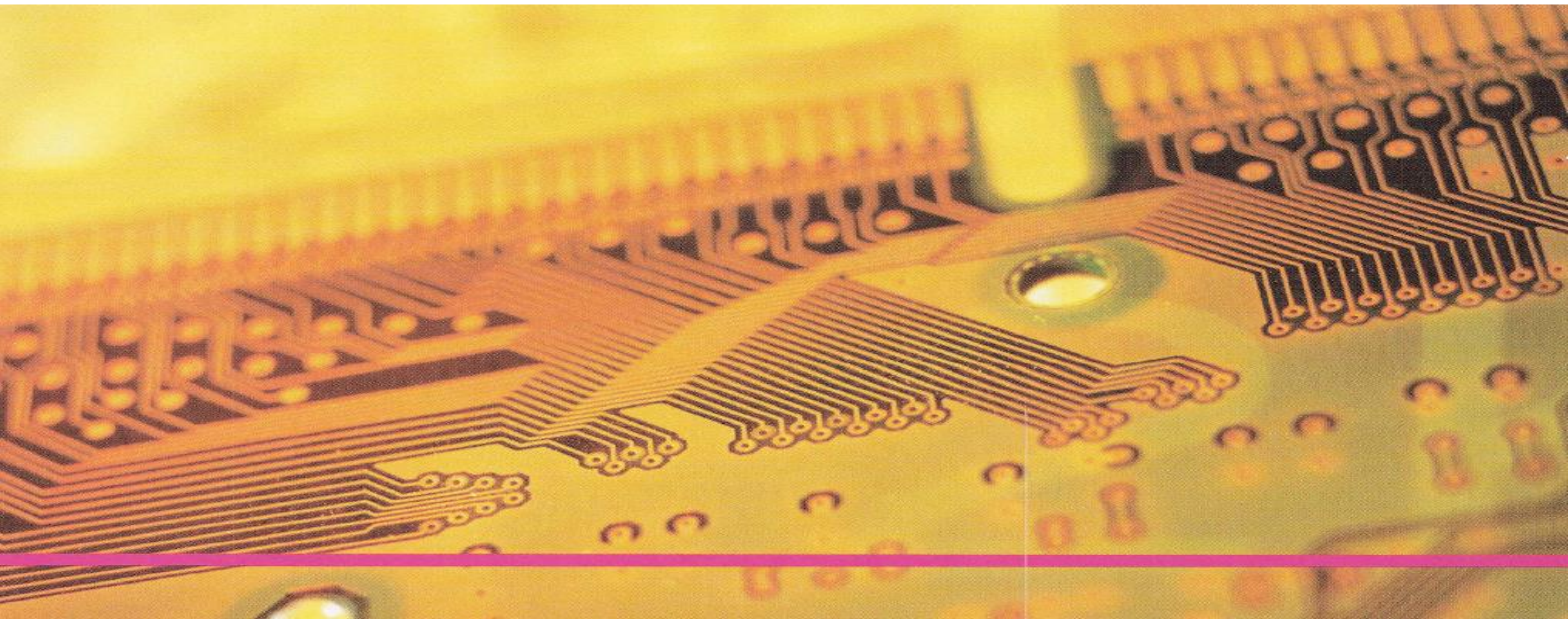
4

BOOLEAN ALGEBRA AND LOGIC SIMPLIFICATION



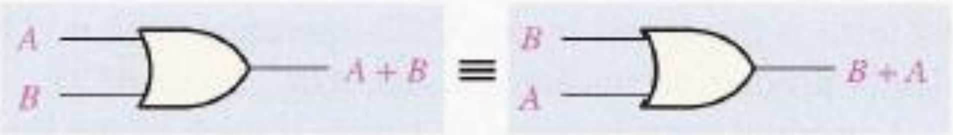
Laws of Boolean Algebra

Commutative Laws The *commutative law of addition* for two variables is written as

$$A + B = B + A$$

FIGURE 4–1

Application of commutative law of addition.

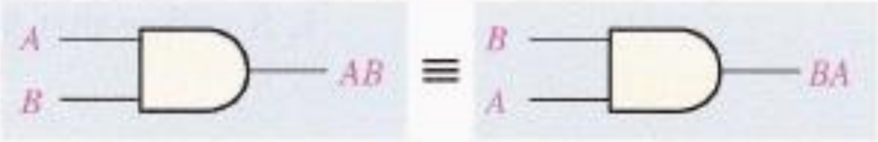


The *commutative law of multiplication* for two variables is

$$AB = BA$$

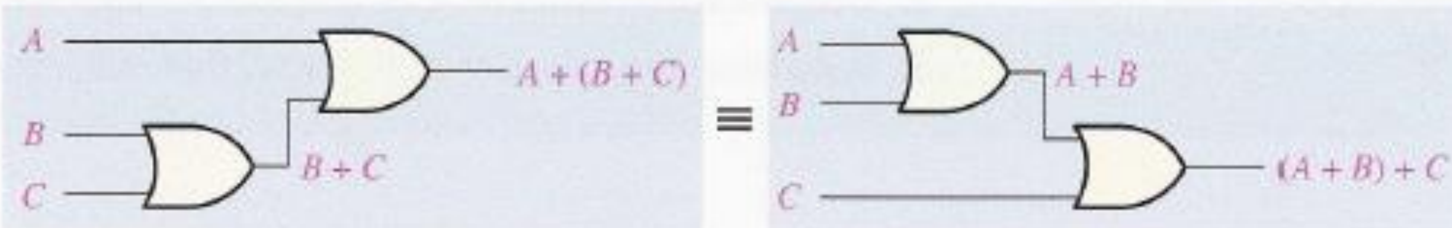
FIGURE 4–2

Application of commutative law of multiplication.



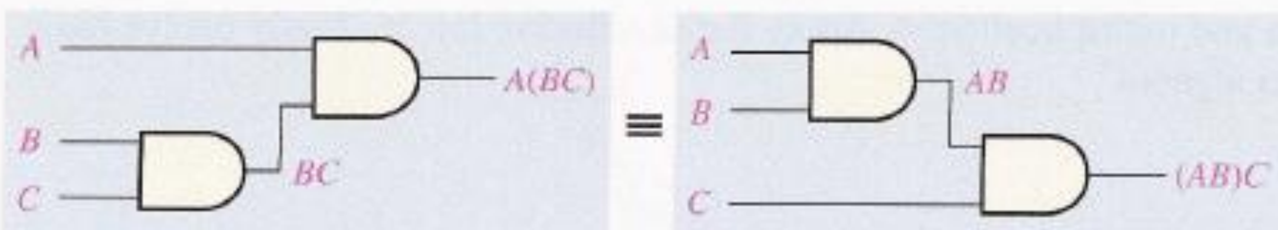
Associative Laws The *associative law of addition* is written as follows for three variables:

$$A + (B + C) = (A + B) + C$$



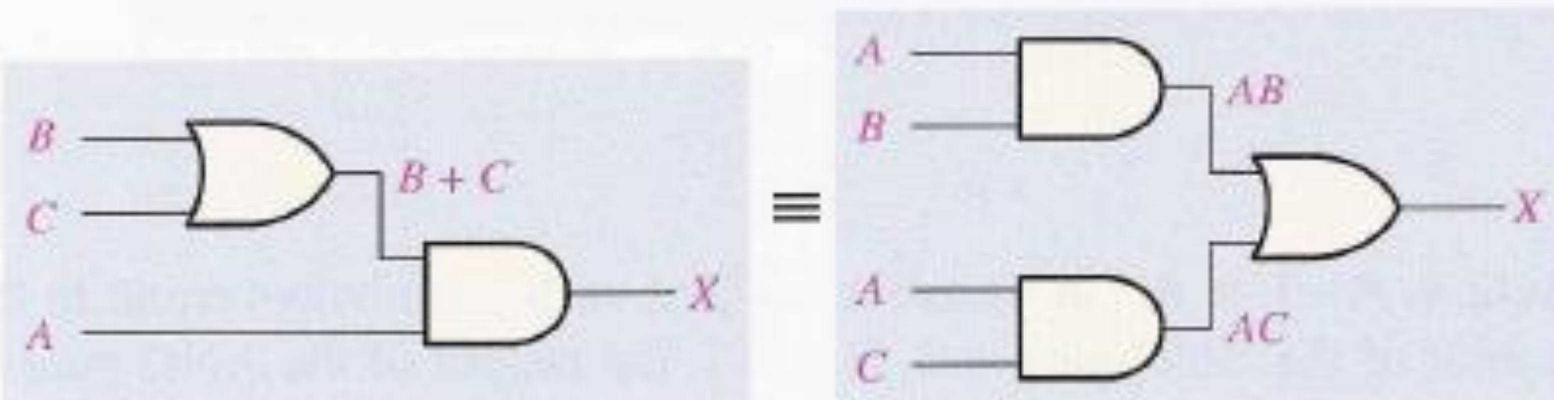
The *associative law of multiplication* is written as follows for three variables:

$$A(BC) = (AB)C$$



Distributive Law The distributive law is written for three variables as follows:

$$A(B + C) = AB + AC$$

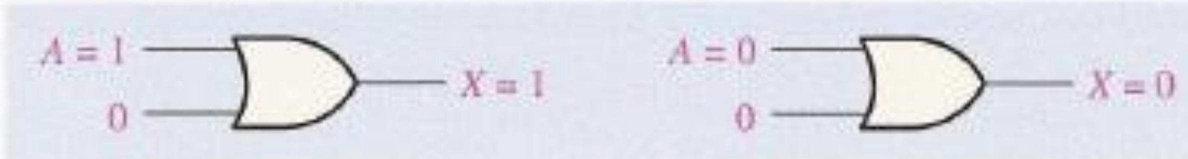


$X = A(B + C)$ $\equiv$ $X = AB + AC$

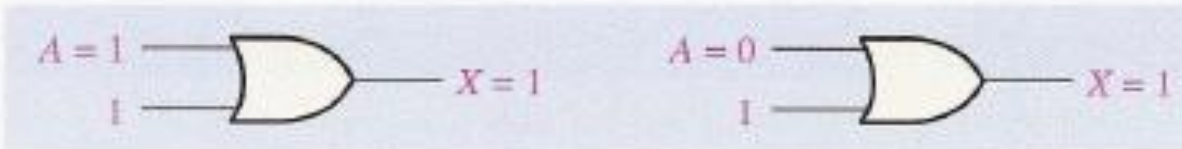
Rules of Boolean Algebra

1. $A + 0 = A$
2. $A + 1 = 1$
3. $A \cdot 0 = 0$
4. $A \cdot 1 = A$
5. $A + A = A$
6. $A + \overline{A} = 1$
7. $A \cdot A = A$
8. $A \cdot \overline{A} = 0$
9. $\overline{\overline{A}} = A$
10. $A + AB = A$
11. $A + \overline{A}B = A + B$
12. $(A + B)(A + C) = A + BC$

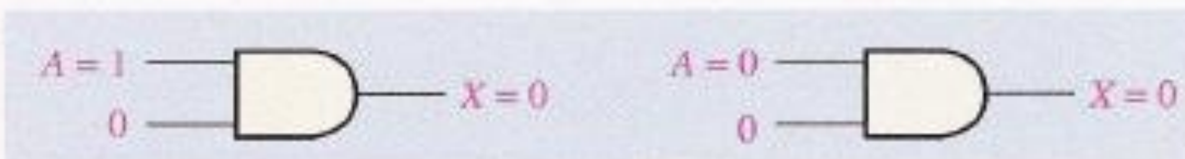
A, *B*, or *C* can represent a single variable or a combination of variables.



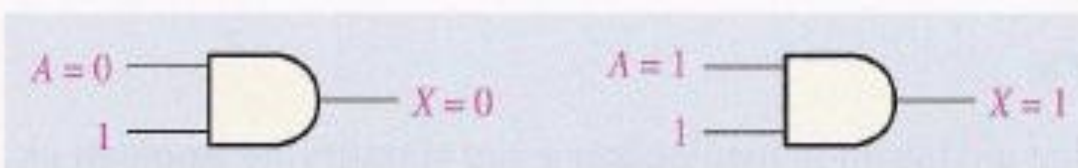
$X = A + 0 = A$



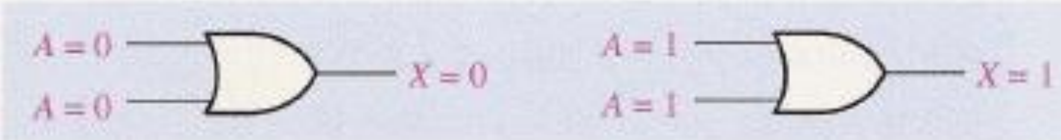
$X = A + 1 = 1$



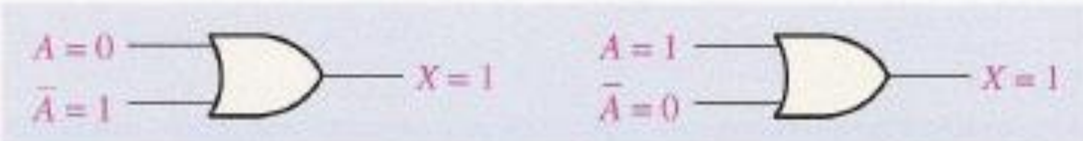
$X = A \cdot 0 = 0$



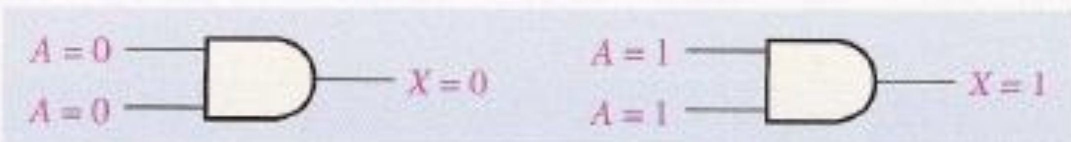
$X = A \cdot 1 = A$



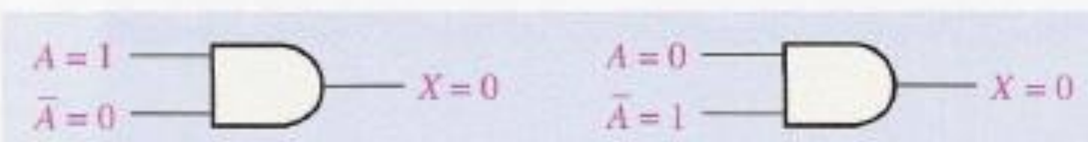
$X = A + A = A$



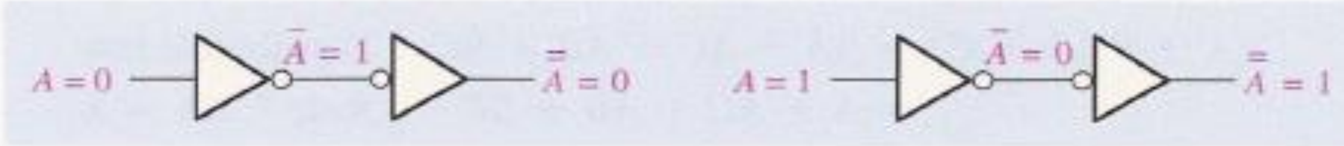
$X = A + \bar{A} = 1$



$X = A \cdot A = A$



$X = A \cdot \bar{A} = 0$



$$\overline{\overline{A}} = A$$

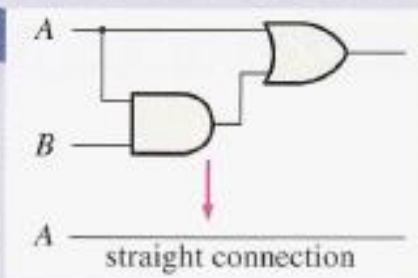
Rule 10. $A + AB = A$ This rule can be proved by applying the distributive law, rule 2, and rule 4 as follows:

$$\begin{aligned} A + AB &= A(1 + B) && \text{Factoring (distributive law)} \\ &= A \cdot 1 && \text{Rule 2: } (1 + B) = 1 \\ &= A && \text{Rule 4: } A \cdot 1 = A \end{aligned}$$

The proof is shown in Table 4–2, which shows the truth table and the resulting logic circuit simplification.

A	B	AB	A + AB
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

equal



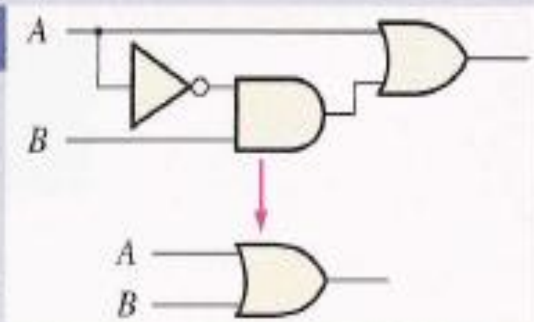
Rule 11. $A + \bar{A}B = A + B$ This rule can be proved as follows:

$$
\begin{aligned}
A + \bar{A}B &= (A + AB) + \bar{A}B && \text{Rule 10: } A = A + AB \\
&= (AA + AB) + \bar{A}B && \text{Rule 7: } A = AA \\
&= AA + AB + A\bar{A} + \bar{A}B && \text{Rule 8: adding } A\bar{A} = 0 \\
&= (A + \bar{A})(A + B) && \text{Factoring} \\
&= 1 \cdot (A + B) && \text{Rule 6: } A + \bar{A} = 1 \\
&= A + B && \text{Rule 4: drop the 1}
\end{aligned}
$$

The proof is shown in Table 4–3, which shows the truth table and the resulting logic circuit simplification.

A	B	$\bar{A}B$	$A + \bar{A}B$	$A + B$
0	0	0	0	0
0	1	1	1	1
1	0	0	1	1
1	1	0	1	1

equal



Rule 12. $(A + B)(A + C) = A + BC$ This rule can be proved as follows:

$$\begin{aligned}
(A + B)(A + C) &= AA + AC + AB + BC && \text{Distributive law} \\
&= A + AC + AB + BC && \text{Rule 7: } AA = A \\
&= A(1 + C) + AB + BC && \text{Factoring (distributive law)} \\
&= A \cdot 1 + AB + BC && \text{Rule 2: } 1 + C = 1 \\
&= A(1 + B) + BC && \text{Factoring (distributive law)} \\
&= A \cdot 1 + BC && \text{Rule 2: } 1 + B = 1 \\
&= A + BC && \text{Rule 4: } A \cdot 1 = A
\end{aligned}$$

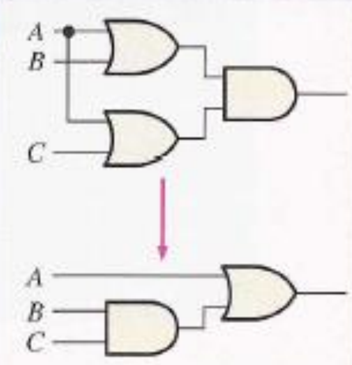
The proof is shown in Table 4–4, which shows the truth table and the resulting logic circuit simplification.

▼ TABLE 4–4

Rule 12: $(A + B)(A + C) = A + BC$. Open file T04-04 to verify.

A	B	C	$A + B$	$A + C$	$(A + B)(A + C)$	BC	$A + BC$
0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	1	1	1	1
1	0	0	1	1	1	0	1
1	0	1	1	1	1	0	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

equal



EXAMPLE 4–8

Using Boolean algebra techniques, simplify this expression:

$$AB + A(B + C) + B(B + C)$$

Solution The following is not necessarily the only approach.

Step 1: Apply the distributive law to the second and third terms in the expression, as follows:

$$AB + AB + AC + BB + BC$$

Step 2: Apply rule 7 ($BB = B$) to the fourth term.

$$AB + AB + AC + B + BC$$

Step 3: Apply rule 5 ($AB + AB = AB$) to the first two terms.

$$AB + AC + B + BC$$

Step 4: Apply rule 10 ($B + BC = B$) to the last two terms.

$$AB + AC + B$$

Step 5: Apply rule 10 ($AB + B = B$) to the first and third terms.

$$B + AC$$

4–3 DEMORGAN'S THEOREMS

One of DeMorgan's theorems is stated as follows:

The complement of a product of variables is equal to the sum of the complements of the variables.

Stated another way,

The complement of two or more ANDed variables is equivalent to the OR of the complements of the individual variables.

The formula for expressing this theorem for two variables is

$$\overline{XY} = \overline{X} + \overline{Y}$$

Equation 4–6

DeMorgan's second theorem is stated as follows:

The complement of a sum of variables is equal to the product of the complements of the variables.

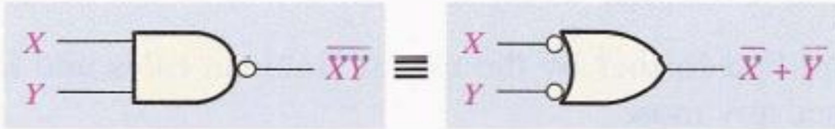
Stated another way,

The complement of two or more ORed variables is equivalent to the AND of the complements of the individual variables.

The formula for expressing this theorem for two variables is

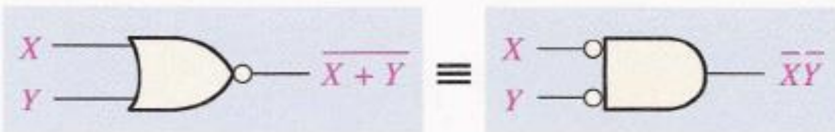
$$\overline{X + Y} = \overline{X}\,\overline{Y}$$

Equation 4–7



NAND　　　　Negative-OR

Inputs		Output	
X	Y	$\overline{XY}$	$\overline{X} + \overline{Y}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0



NOR　　　　Negative-AND

Inputs		Output	
X	Y	$\overline{X + Y}$	$\overline{X}\,\overline{Y}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

As stated, DeMorgan's theorems also apply to expressions in which there are more than two variables. The following examples illustrate the application of DeMorgan's theorems to 3-variable and 4-variable expressions.

EXAMPLE 4–3

Apply DeMorgan's theorems to the expressions $\overline{XYZ}$ and $\overline{X + Y + Z}$.

Solution

$$\overline{XYZ} = \overline{X} + \overline{Y} + \overline{Z}$$

$$\overline{X + Y + Z} = \overline{X}\,\overline{Y}\,\overline{Z}$$

Related Problem Apply DeMorgan's theorem to the expression $\overline{\overline{X} + \overline{Y} + \overline{Z}}$.

EXAMPLE 4–6

Apply DeMorgan's theorems to each expression:

(a) $\overline{\overline{(A + B)} + \overline{C}}$ **(b)** $\overline{(\overline{A} + B) + CD}$ **(c)** $\overline{(A + B)\overline{C}\,\overline{D} + E + \overline{F}}$

Solution **(a)** $\overline{\overline{(A + B)} + \overline{C}} = (\overline{\overline{A + B}})\overline{\overline{C}} = (A + B)C$

(b) $\overline{(\overline{A} + B) + CD} = (\overline{\overline{A} + B})\overline{CD} = (\overline{\overline{A}}\,\overline{B})(\overline{C} + \overline{D}) = A\overline{B}(\overline{C} + \overline{D})$

(c) $\overline{(A + B)\overline{C}\,\overline{D} + E + \overline{F}} = (\overline{(A + B)\overline{C}\,\overline{D}})(\overline{E + \overline{F}}) = (\overline{A}\,\overline{B} + C + D)\overline{E}F$

Related Problem Apply DeMorgan's theorems to the expression $\overline{\overline{A}B(C + \overline{D}) + E}$.

EXAMPLE 4–11

Simplify the following Boolean expression:

$$\overline{AB + AC} + \overline{A}\,\overline{B}C$$

Solution **Step 1:** Apply DeMorgan's theorem to the first term.

$$(\overline{AB})(\overline{AC}) + \overline{A}\,\overline{B}C$$

Step 2: Apply DeMorgan's theorem to each term in parentheses.

$$(\overline{A} + \overline{B})(\overline{A} + \overline{C}) + \overline{A}\,\overline{B}C$$

Step 3: Apply the distributive law to the two terms in parentheses.

$$\overline{A}\,\overline{A} + \overline{A}\,\overline{C} + \overline{A}\,\overline{B} + \overline{B}\,\overline{C} + \overline{A}\,\overline{B}C$$

Step 4: Apply rule 7 ($\overline{A}\,\overline{A} = \overline{A}$) to the first term, and apply rule 10 $[\overline{A}\,\overline{B} + \overline{A}\,\overline{B}C = \overline{A}\,\overline{B}(1 + C) = \overline{A}\,\overline{B}]$ to the third and last terms.

$$\overline{A} + \overline{A}\,\overline{C} + \overline{A}\,\overline{B} + \overline{B}\,\overline{C}$$

Step 5: Apply rule 10 $[\overline{A} + \overline{A}\,\overline{C} = \overline{A}(1 + \overline{C}) = \overline{A}]$ to the first and second terms.

$$\overline{A} + \overline{A}\,\overline{B} + \overline{B}\,\overline{C}$$

Step 6: Apply rule 10 $[\overline{A} + \overline{A}\,\overline{B} = \overline{A}(1 + \overline{B}) = \overline{A}]$ to the first and second terms.

$$\overline{A} + \overline{B}\,\overline{C}$$

Related Problem Simplify the Boolean expression $\overline{AB} + \overline{AC} + \overline{A}\,\overline{B}\,\overline{C}$.

H.W

SECTION 4-3 REVIEW

1. Apply DeMorgan's theorems to the following expressions:

   (a) $\overline{ABC} + \overline{(\overline{D} + E)}$ (b) $\overline{(A + B)C}$ (c) $\overline{A + B + C} + \overline{\overline{D}E}$

SECTION 4-5 REVIEW

1. Simplify the following Boolean expressions if possible:

   (a) $A + AB + A\overline{B}C$ (b) $(\overline{A} + B)C + ABC$ (c) $A\overline{B}C(BD + CDE) + A\overline{C}$

4–8 THE KARNAUGH MAP

A Karnaugh map provides a systematic method for simplifying Boolean expressions

A **Karnaugh map** is similar to a truth table because it presents all of the possible values of input variables and the resulting output for each value. Instead of being organized into columns and rows like a truth table, the Karnaugh map is an array of **cells** in which each cell represents a binary value of the input variables. The cells are arranged in a way so that simplification of a given expression is simply a matter of properly grouping the cells. Karnaugh maps can be used for expressions with two, three, four, and five variables, but we will discuss

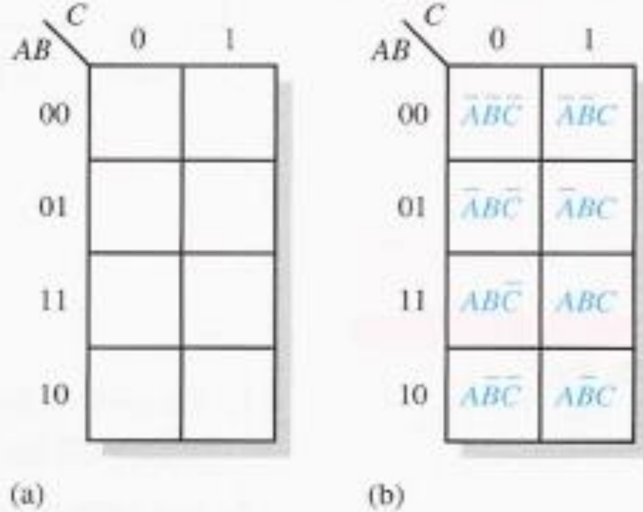
The number of cells in a Karnaugh map is equal to the total number of possible input variable combinations as is the number of rows in a truth table. For three variables, the number of cells is $2^3 = 8$. For four variables, the number of cells is $2^4 = 16$.

The 3-Variable Karnaugh Map

The 3-variable Karnaugh map is an array of eight cells, as shown in Figure 4–21(a). In this case, *A*, *B*, and *C* are used for the variables although other letters could be used. Binary values of *A* and *B* are along the left side (notice the sequence) and the values of *C* are across the top. The value of a given cell is the binary values of *A* and *B* at the left in the same row combined with the value of *C* at the top in the same column. For example, the cell in the upper left corner has a binary value of 000 and the cell in the lower right corner has a binary value of 101. Figure 4–21(b) shows the standard product terms that are represented by each cell in the Karnaugh map.

FIGURE 4–21

A 3-variable Karnaugh map showing product terms.



The 4-Variable Karnaugh Map

The 4-variable Karnaugh map is an array of sixteen cells, as shown in Figure 4–22(a). Binary values of A and B are along the left side and the values of C and D are across the top. The value of a given cell is the binary values of A and B at the left in the same row com-

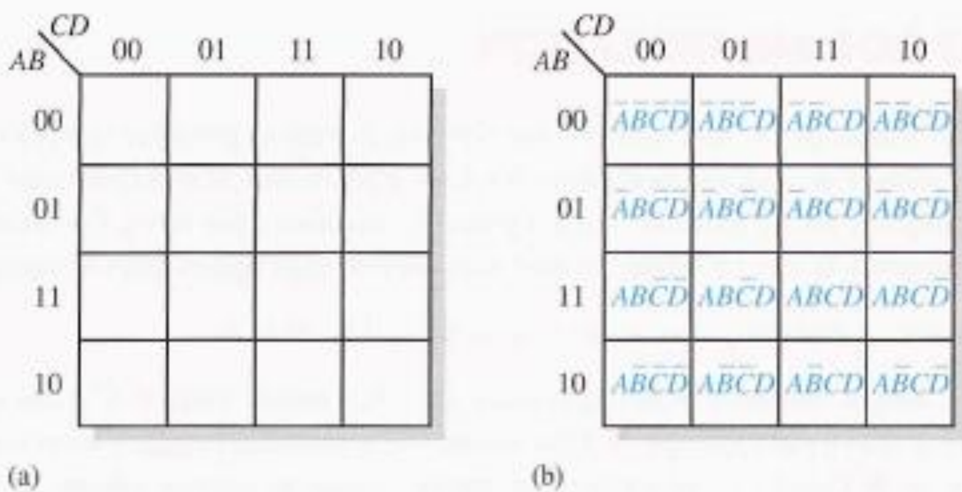


FIGURE 4–22

A 4-variable Karnaugh map.

bined with the binary values of C and D at the top in the same column. For example, the cell in the upper right corner has a binary value of 0010 and the cell in the lower right corner has a binary value of 1010. Figure 4–22(b) shows the standard product terms that are represented by each cell in the 4-variable Karnaugh map.

Mapping a Standard SOP Expression

For an SOP expression in standard form, a 1 is placed on the Karnaugh map for each product term in the expression. Each 1 is placed in a cell corresponding to the value of a product term. For example, for the product term $A\overline{B}C$, a 1 goes in the 101 cell on a 3-variable map.

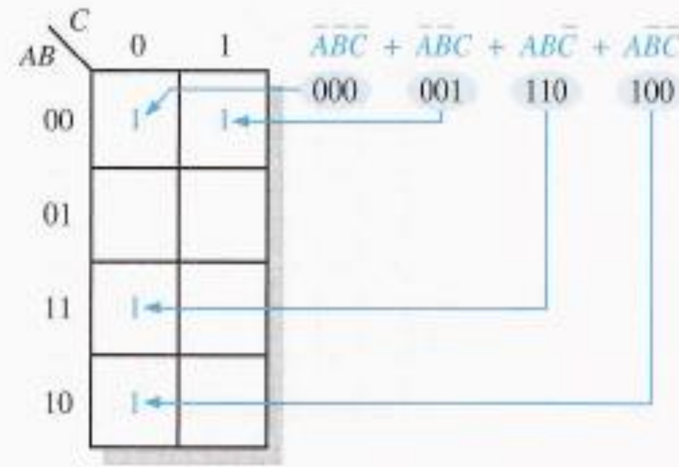
When an SOP expression is completely mapped, there will be a number of 1s on the Karnaugh map equal to the number of product terms in the standard SOP expression. The cells that do not have a 1 are the cells for which the expression is 0. Usually, when working with SOP expressions, the 0s are left off the map. The following steps and the illustration in Figure 4–24 show the mapping process.

Step 1. Determine the binary value of each product term in the standard SOP expression. After some practice, you can usually do the evaluation of terms mentally.

Step 2. As each product term is evaluated, place a 1 on the Karnaugh map in the cell having the same value as the product term.

FIGURE 4–24

Example of mapping a standard SOP expression.



EXAMPLE 4–21

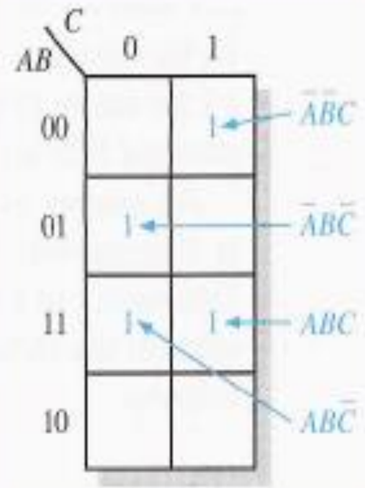
Map the following standard SOP expression on a Karnaugh map:

$$\overline{A}\,\overline{B}C + \overline{A}B\overline{C} + AB\overline{C} + ABC$$

Solution Evaluate the expression as shown below. Place a 1 on the 3-variable Karnaugh map in Figure 4–25 for each standard product term in the expression.

$$\begin{array}{cccc} \overline{A}\,\overline{B}C + & \overline{A}B\overline{C} + & AB\overline{C} + & ABC \\ 001 & 010 & 110 & 111 \end{array}$$

FIGURE 4–25



Related Problem Map the standard SOP expression $\overline{A}BC + A\overline{B}C + A\overline{B}\,\overline{C}$ on a Karnaugh map.

EXAMPLE 4–22

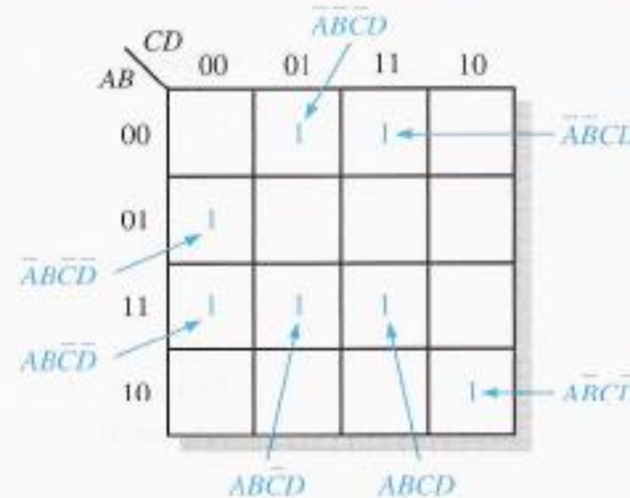
Map the following standard SOP expression on a Karnaugh map:

$$\overline{A}\,\overline{B}CD + \overline{A}B\overline{C}\,\overline{D} + AB\overline{C}D + ABCD + AB\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + A\overline{B}C\overline{D}$$

Solution Evaluate the expression as shown below. Place a 1 on the 4-variable Karnaugh map in Figure 4–26 for each standard product term in the expression.

$$\underset{0011}{\overline{A}\,\overline{B}CD} + \underset{0100}{\overline{A}B\overline{C}\,\overline{D}} + \underset{1101}{AB\overline{C}D} + \underset{1111}{ABCD} + \underset{1100}{AB\overline{C}\,\overline{D}} + \underset{0001}{\overline{A}\,\overline{B}\,\overline{C}D} + \underset{1010}{A\overline{B}C\overline{D}}$$

FIGURE 4–26



Related Problem Map the following standard SOP expression on a Karnaugh map:

$$\overline{A}BC\overline{D} + ABC\overline{D} + AB\overline{C}\,\overline{D} + ABCD$$

Karnaugh Map Simplification of SOP Expressions

The process that results in an expression containing the fewest possible terms with the fewest possible variables is called **minimization.** After an SOP expression has been mapped, a minimum SOP expression is obtained by grouping the 1s and determining the minimum SOP expression from the map.

Grouping the 1s You can group 1s on the Karnaugh map according to the following rules by enclosing those adjacent cells containing 1s. The goal is to maximize the size of the groups and to minimize the number of groups.

1. A group must contain either 1, 2, 4, 8, or 16 cells, which are all powers of two. In the case of a 3-variable map, $2^3 = 8$ cells is the maximum group.
2. Each cell in a group must be adjacent to one or more cells in that same group, but all cells in the group do not have to be adjacent to each other.
3. Always include the largest possible number of 1s in a group in accordance with rule 1.
4. Each 1 on the map must be included in at least one group. The 1s already in a group can be included in another group as long as the overlapping groups include noncommon 1s.

EXAMPLE 4–25

Group the 1s in each of the Karnaugh maps in Figure 4–29.

(a)

AB \ C	0	1
00	1	
01		1
11	1	1
10		

(b)

AB \ C	0	1
00	1	1
01	1	
11		1
10	1	1

(c)

AB \ CD	00	01	11	10
00	1	1		
01	1	1	1	1
11				
10		1	1	

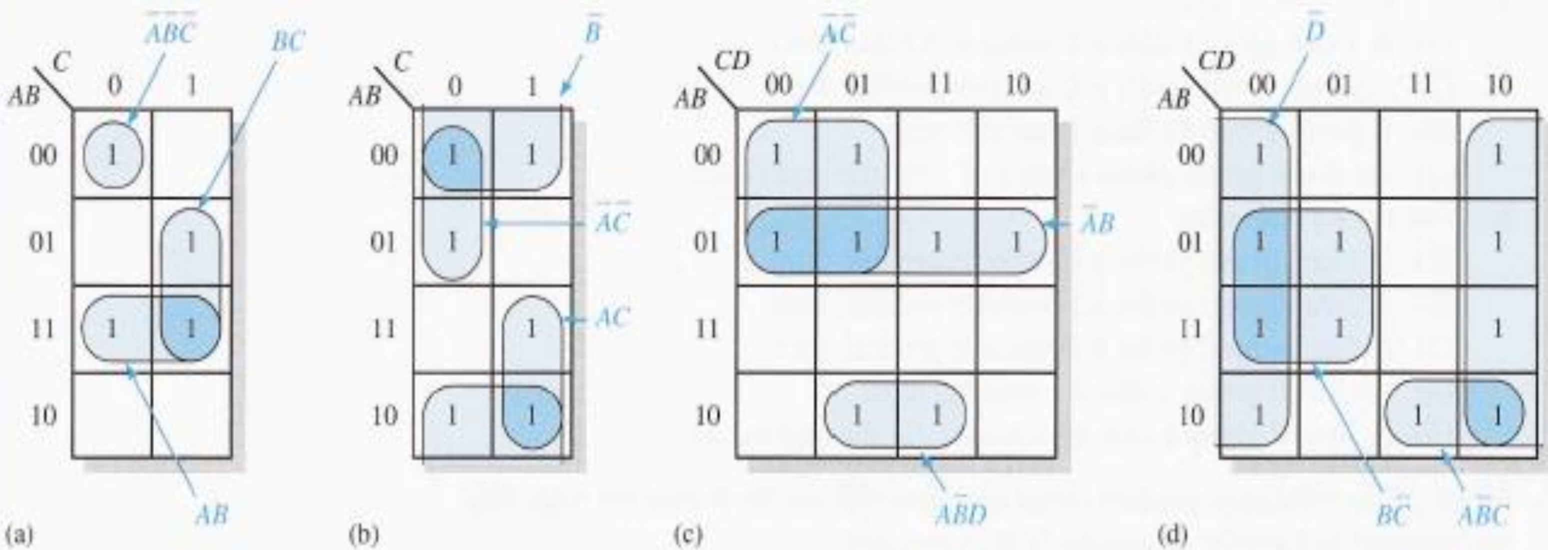
(d)

AB \ CD	00	01	11	10
00	1			1
01	1	1		1
11	1	1		1
10	1		1	1

▲ FIGURE 4–29

EXAMPLE 4–27

Determine the product terms for each of the Karnaugh maps in Figure 4–32 and write the resulting minimum SOP expression.



▲ **FIGURE 4–32**

Solution The resulting minimum product term for each group is shown in Figure 4–32. The minimum SOP expressions for each of the Karnaugh maps in the figure are

(a) $AB + BC + \overline{A}\,\overline{B}\,\overline{C}$ **(b)** $\overline{B} + \overline{A}\,\overline{C} + AC$

(c) $\overline{A}B + \overline{A}\,\overline{C} + A\overline{B}D$ **(d)** $\overline{D} + A\overline{B}C + B\overline{C}$

Related Problem For the Karnaugh map in Figure 4–32(d), add a 1 in the 0111 cell and determine the resulting SOP expression.

EXAMPLE 4–28

Use a Karnaugh map to minimize the following standard SOP expression:

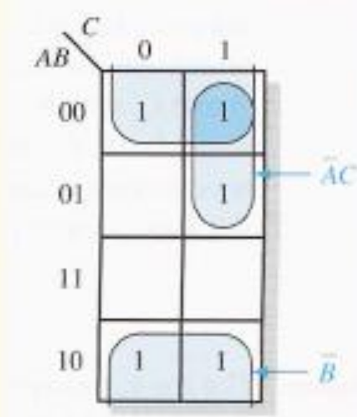
$$A\overline{B}C + \overline{A}BC + \overline{A}\,\overline{B}C + \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}\,\overline{C}$$

Solution The binary values of the expression are

$$101 + 011 + 011 + 000 + 100$$

Map the standard SOP expression and group the cells as shown in Figure 4–33.

FIGURE 4–33



for each group is shown. The resulting minimum SOP expression is

$$\overline{B} + \overline{A}C$$

Keep in mind that this minimum expression is equivalent to the original standard expression.

Related Problem Use a Karnaugh map to simplify the following standard SOP expression:

$$X\overline{Y}Z + XY\overline{Z} + \overline{X}YZ + \overline{X}Y\overline{Z} + X\overline{Y}\,\overline{Z} + XYZ$$

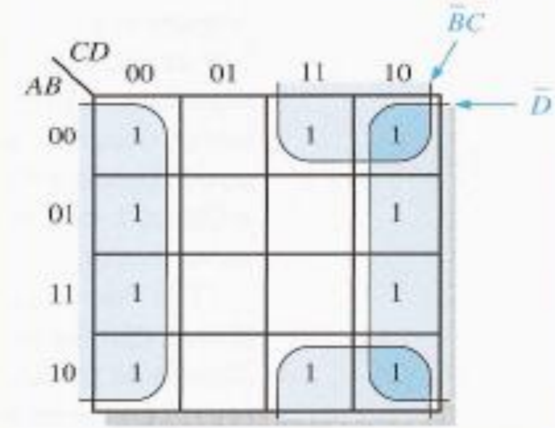
EXAMPLE 4–29

Use a Karnaugh map to minimize the following SOP expression:

$$\overline{B}\,\overline{C}\,\overline{D} + \overline{A}B\overline{C}\,\overline{D} + AB\overline{C}\,\overline{D} + \overline{A}\,\overline{B}CD + A\overline{B}CD + \overline{A}\,\overline{B}C\overline{D} + \overline{A}BC\overline{D} + ABC\overline{D} + A\overline{B}C\overline{D}$$

Solution The first term $\overline{B}\,\overline{C}\,\overline{D}$ must be expanded into $A\overline{B}\,\overline{C}\,\overline{D}$ and $\overline{A}\,\overline{B}\,\overline{C}\,\overline{D}$ to get the standard SOP expression, which is then mapped; and the cells are grouped as shown in Figure 4–34.

▶ FIGURE 4–34



Notice that both groups exhibit "wrap around" adjacency. The group of eight is formed because the cells in the outer columns are adjacent. The group of four is formed to pick up the remaining two 1s because the top and bottom cells are adjacent. The product term for each group is shown. The resulting minimum SOP expression is

$$\overline{D} + \overline{B}C$$

Keep in mind that this minimum expression is equivalent to the original standard expression.

Related Problem Use a Karnaugh map to simplify the following SOP expression:

$$\overline{W}\,\overline{X}\,\overline{Y}\,\overline{Z} + W\overline{X}YZ + W\overline{X}\,\overline{Y}Z + \overline{W}YZ + W\overline{X}\,\overline{Y}\,\overline{Z}$$

Mapping a Standard POS Expression

For a POS expression in standard form, a 0 is placed on the Karnaugh map for each sum term in the expression. Each 0 is placed in a cell corresponding to the value of a sum term. For example, for the sum term $A + \overline{B} + C$, a 0 goes in the 010 cell on a 3-variable map.

When a POS expression is completely mapped, there will be a number of 0s on the Karnaugh map equal to the number of sum terms in the standard POS expression. The cells that do not have a 0 are the cells for which the expression is 1. Usually, when working with POS expressions, the 1s are left off. The following steps and the illustration in Figure 4–37 show the mapping process.

Step 1. Determine the binary value of each sum term in the standard POS expression. This is the binary value that makes the term equal to 0.

Step 2. As each sum term is evaluated, place a 0 on the Karnaugh map in the corresponding cell.

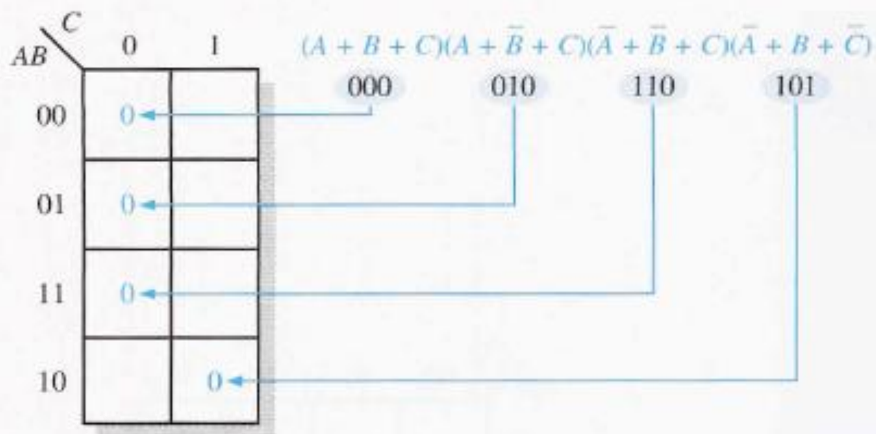


FIGURE 4–37

Example of mapping a standard POS expression.

EXAMPLE 4–30

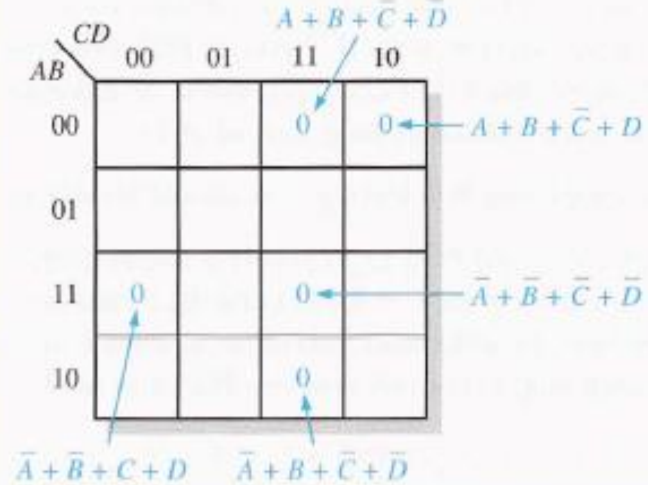
Map the following standard POS expression on a Karnaugh map:

$$(\overline{A} + \overline{B} + C + D)(\overline{A} + B + \overline{C} + \overline{D})(A + B + \overline{C} + D)(\overline{A} + \overline{B} + \overline{C} + \overline{D})(A + B + \overline{C} + \overline{D})$$

Solution Evaluate the expression as shown below and place a 0 on the 4-variable Karnaugh map in Figure 4–38 for each standard sum term in the expression.

$$\underbrace{(\overline{A} + \overline{B} + C + D)}_{1100}\underbrace{(\overline{A} + B + \overline{C} + \overline{D})}_{1011}\underbrace{(A + B + \overline{C} + D)}_{0010}\underbrace{(\overline{A} + \overline{B} + \overline{C} + \overline{D})}_{1111}\underbrace{(A + B + \overline{C} + \overline{D})}_{0011}$$

FIGURE 4–38



Related Problem Map the following standard POS expression on a Karnaugh map:

$$(A + \overline{B} + \overline{C} + D)(A + B + C + \overline{D})(A + B + C + D)(\overline{A} + B + \overline{C} + D)$$

Karnaugh Map Simplification of POS Expressions

EXAMPLE 4–31

Use a Karnaugh map to minimize the following standard POS expression:

$$(A + B + C)(A + B + \overline{C})(A + \overline{B} + C)(A + \overline{B} + \overline{C})(\overline{A} + \overline{B} + C)$$

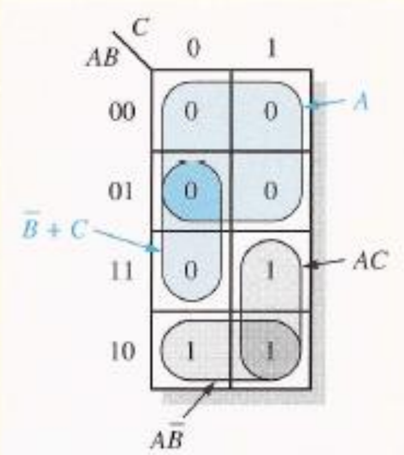
Also, derive the equivalent SOP expression.

Solution The combinations of binary values of the expression are

$$(0 + 0 + 0)\,(0 + 0 + 1)\,(0 + 1 + 0)\,(0 + 1 + 1)\,(1 + 1 + 0)$$

Map the standard POS expression and group the cells as shown in Figure 4–39.

▶ **FIGURE 4–39**



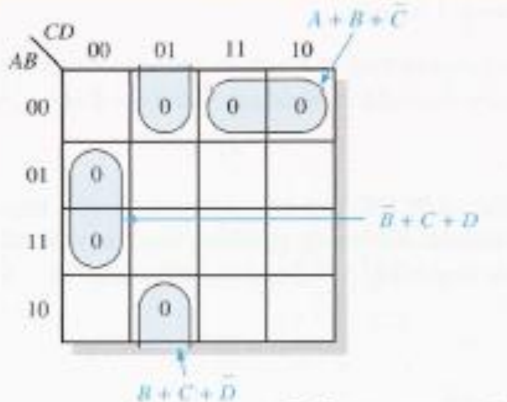
Notice how the 0 in the 110 cell is included into a 2-cell group by utilizing the 0 in the 4-cell group. The sum term for each blue group is shown in the figure and the resulting minimum POS expression is

$$A(\overline{B} + C)$$

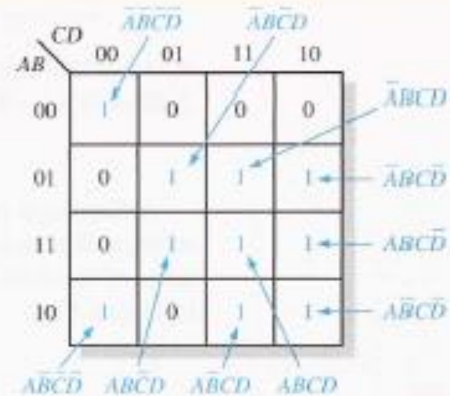
EXAMPLE 4–33

Using a Karnaugh map, convert the following standard POS expression into a minimum POS expression, a standard SOP expression, and a minimum SOP expression.

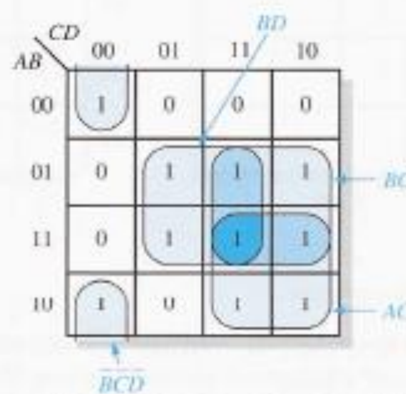
$$(\bar{A} + \bar{B} + C + D)(A + \bar{B} + C + D)(A + B + C + \bar{D})$$
$$(A + B + \bar{C} + \bar{D})(\bar{A} + B + C + \bar{D})(A + B + \bar{C} + D)$$



(a) Minimum POS: $(A + B + C)(\bar{B} + \bar{C} + D)(B + C + \bar{D})$



(b) Standard SOP:
$\bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}B\bar{C}D + \bar{A}BCD + \bar{A}BC\bar{D} + ABC\bar{D} + A\bar{B}C\bar{D} +$
$A\bar{B}\bar{C}\bar{D} + AB\bar{C}D + A\bar{B}CD + ABCD$



(c) Minimum SOP: $AC + BC + BD + \bar{B}\bar{C}\bar{D}$

"Don't Care" Conditions

Sometimes a situation arises in which some input variable combinations are not allowed. For example, recall that in the BCD code covered in Chapter 2, there are six invalid combinations: 1010, 1011, 1100, 1101, 1110, and 1111. Since these unallowed states will never occur in an application involving the BCD code, they can be treated as **"don't care"** terms with respect to their effect on the output. That is, for these "don't care" terms either a 1 or a 0 may be assigned to the output; it really does not matter since they will never occur.

The "don't care" terms can be used to advantage on the Karnaugh map. Figure 4–36 shows that for each "don't care" term, an X is placed in the cell. When grouping the 1s, the Xs can be treated as 1s to make a larger grouping or as 0s if they cannot be used to advantage. The larger a group, the simpler the resulting term will be.

The truth table in Figure 4–36(a) describes a logic function that has a 1 output only when the BCD code for 7, 8, or 9 is present on the inputs. If the "don't cares" are used as 1s, the resulting expression for the function is $A + BCD$, as indicated in part (b). If the "don't cares" are not used as 1s, the resulting expression is $A\overline{B}\,\overline{C} + \overline{A}BCD$; so you can see the advantage of using "don't care" terms to get the simplest expression.

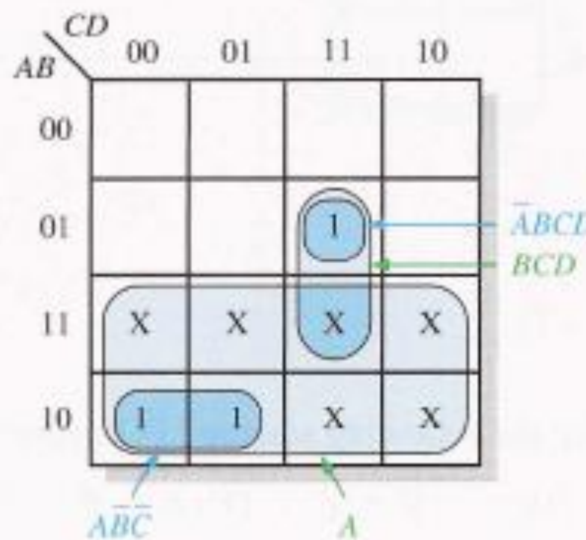
FIGURE 4–36

Example of the use of "don't care" conditions to simplify an expression.

Inputs $ABCD$	Output Y
0 0 0 0	0
0 0 0 1	0
0 0 1 0	0
0 0 1 1	0
0 1 0 0	0
0 1 0 1	0
0 1 1 0	0
0 1 1 1	1
1 0 0 0	1
1 0 0 1	1
1 0 1 0	X
1 0 1 1	X
1 1 0 0	X
1 1 0 1	X
1 1 1 0	X
1 1 1 1	X

Don't cares

(a) Truth table



(b) Without "don't cares" $Y = A\bar{B}\bar{C} + \bar{A}BCD$
With "don't cares" $Y = A + BCD$


THANK
YOU