Continuity

Def:- A function f is Continuous at a number "a" if

$$\lim_{x \to a} f(x) = f(a)$$

this mean the following

① $f(a)$ is defined. (i.e $a \in D_f$)

② $\lim_{x \to a} f(x)$ exists.

③ $\lim_{x \to a} f(x) = f(a)$

* If f is Continuous at every point in the domain, then say that f is Continuous

* If a function f is not continuous at a, then we say that f is discontinuous at "a" & "a" is a point of discontinuity of f

e.g $f(x) = x+1$ if $(x=0)$

① $f(0) = 0+1 = 1$

② $\lim_{x \to 0} f(x) = \lim_{x \to 0} x+1 = 0+1 = 1$ exist

③ $\lim_{x \to 0} f(x) = f(a) = 1$

$\therefore f$ is Continuous at $x=0$

e.g $f(x) = \frac{1}{x}$ (at $x=0$)

① $f(0) = \frac{1}{0}$ undefined (not exists)

$\therefore f$ is not ~~continuous~~ continuous at 0

e.g let $f(x) = \begin{cases} \frac{x^2-1}{x-1} & x \neq 1 \\ 5 & x = 1 \end{cases}$

test for Continuity at $x=1$?

① $f(1) = 5$

② $\lim_{x\to 1} f(x) = \lim_{x\to 1} \frac{x^2-1}{x-1} = \lim_{x\to 1} \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}} = \lim_{x\to 1} x+1 = \boxed{2}$

③ $\lim_{x\to 1} f(x) \neq f(1) \Rightarrow f$ is not continuous at 1

H.W $f(x) = \begin{cases} \frac{\sin x}{x} & x \neq 0 \\ 1 & x = 0 \end{cases}$

Test for continuity at $x=0$

properties of continuous :- let f, g be Continuous functions at "a" then the following are cont. at "a"

① $f \mp g$

② $c.f$, for any constant c.

③ $f.g$

④ f/g , if $g(a) \neq 0$

e.g

$$f(x) = \begin{cases} 2x-1 & x<1 \\ \dfrac{3x+2}{x^2-4} & x \geq 1 \end{cases}$$

test for continuity at $x=1$?

note $x \to 1^+ \overset{\text{means}}{\Longrightarrow} x \to 1$ and $x>1$ &

$x \to 1^- \overset{\text{means}}{\Longrightarrow} x \to 1$ & $x<1$

① $f(1) = \frac{-5}{3}$

② ⓐ $\lim\limits_{x \to 1^+} f(x) = \lim\limits_{x \to 1^+} \frac{3x+2}{x^2-4} = \frac{-5}{3}$

ⓑ $\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^-} 2x-1 = 1$

Since $a \neq b \Rightarrow \lim\limits_{x \to 1}$ does not exist

So $f(x)$ is not continuous at $x=1$

e.g $f(x) = |2x-1|$ test for continuity at $x=\frac{1}{2}$

$$f(x) = \begin{cases} 2x-1 & x \geq \frac{1}{2} \\ -(2x-1) & x < \frac{1}{2} \end{cases}$$

① $f(\frac{1}{2}) = 2(\frac{1}{2})-1 = 0$

② ⓐ $\lim\limits_{x \to \frac{1}{2}^+} f(x) = \lim\limits_{x \to \frac{1}{2}^+} 2(\frac{1}{2})-1 = 0$

(b) $\lim_{x \to \frac{1}{2}^-} f(x) = \lim_{x \to \frac{1}{2}^-} (-2x+1) = -2(\frac{1}{2})+1 = 0$

$\because a=b \quad \therefore \lim_{x \to \frac{1}{2}} f(x)$ exists

(3) Since $f(\frac{1}{2}) = \lim_{x \to \frac{1}{2}} f(x)$

$\therefore f$ is continuous at $x = \frac{1}{2}$

e.g