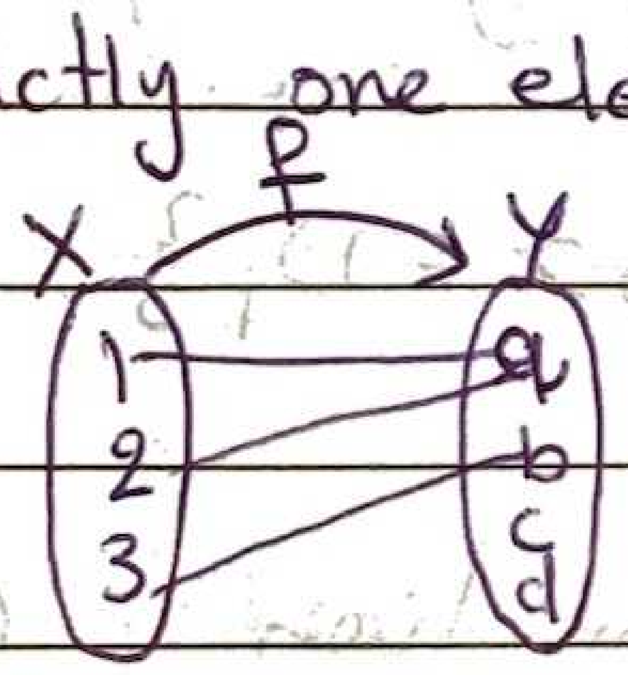
H.W ① $\frac{1}{(6-x)} \geq \frac{2}{(x+4)}$

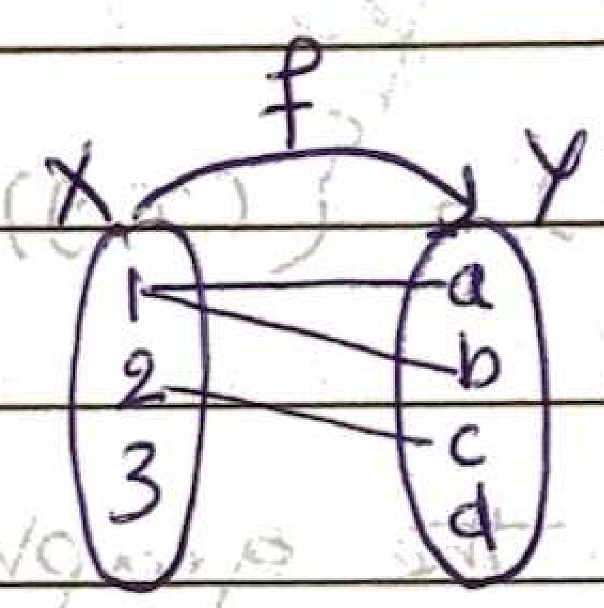
② $\frac{2}{6x-3} \leq 1$ ③ $\frac{1-x}{x} > 1$

chapter two

Functions and Graphs

Def. A Function f is a relation between two sets X and Y such that each element in X has exactly one element in Y.





f is a function

f is not a function because 1 in X goes to two element a and b in Y

* $f : X \to Y$ (f is a function from X to Y)

* The set X is called the domain of the function f and it is denoted by D_f. The number $f(x)$ is the value of f at x

* ~~The range~~ The set Y is called the Co-domain of the function, the range of f is the set of all possible values of $f(x)$ as x varies

throughout the domain and it is denoted by R_f,

* A point x in D_f is Called an independent Varible, and a point y in R_f is Called a dependent Varible (y depends on x)

Graphs

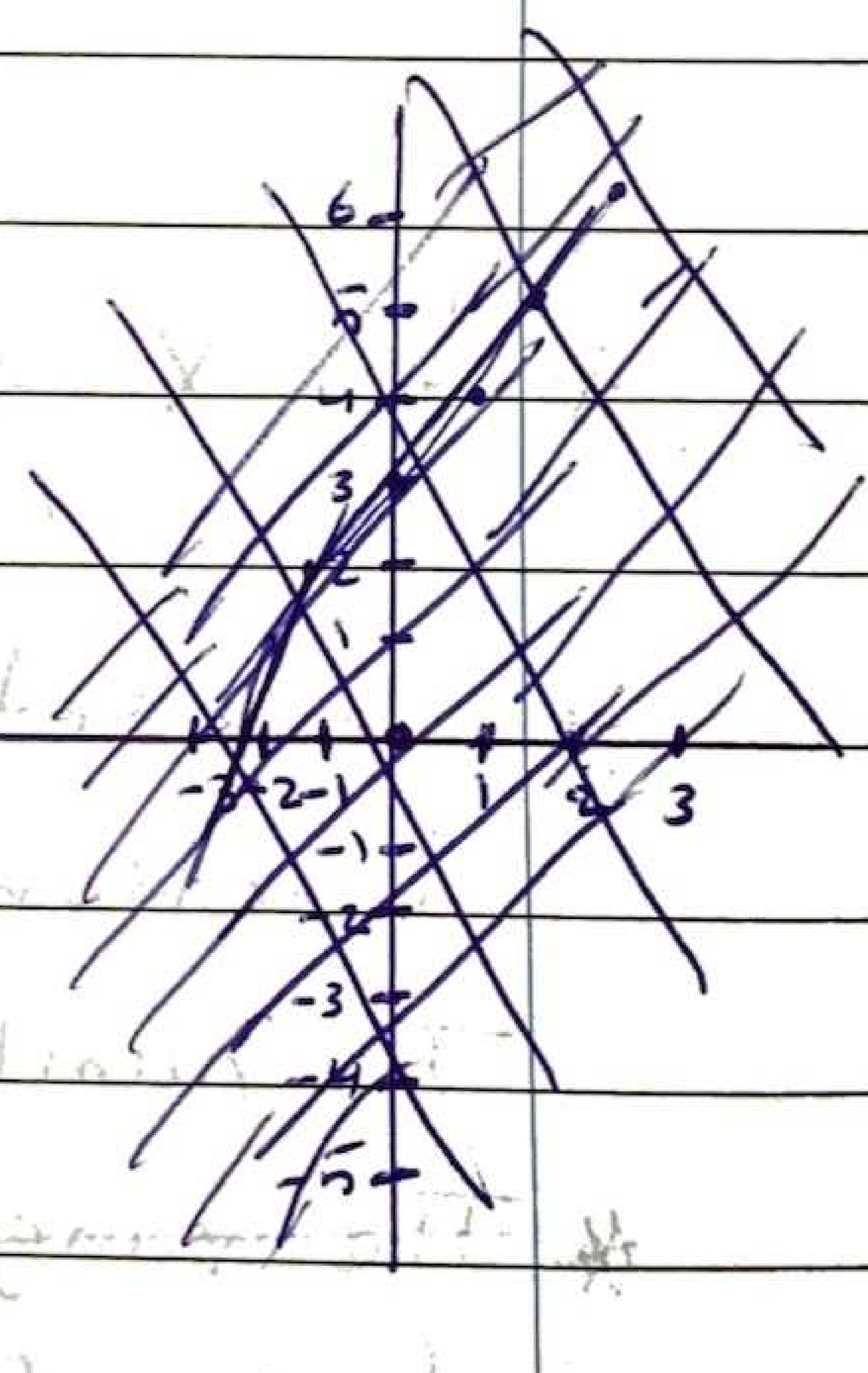
If f is a function and $y = f(x)$ for each x in the domain D_f, we call the set of ~~orderd~~ ordered pairs (x, y) the graph of f.

$$\text{graph}(f) = \{(x,y) : y = f(x), x \in D_f\}$$

e.g find the graph of the function $f(x) = 3 + x$

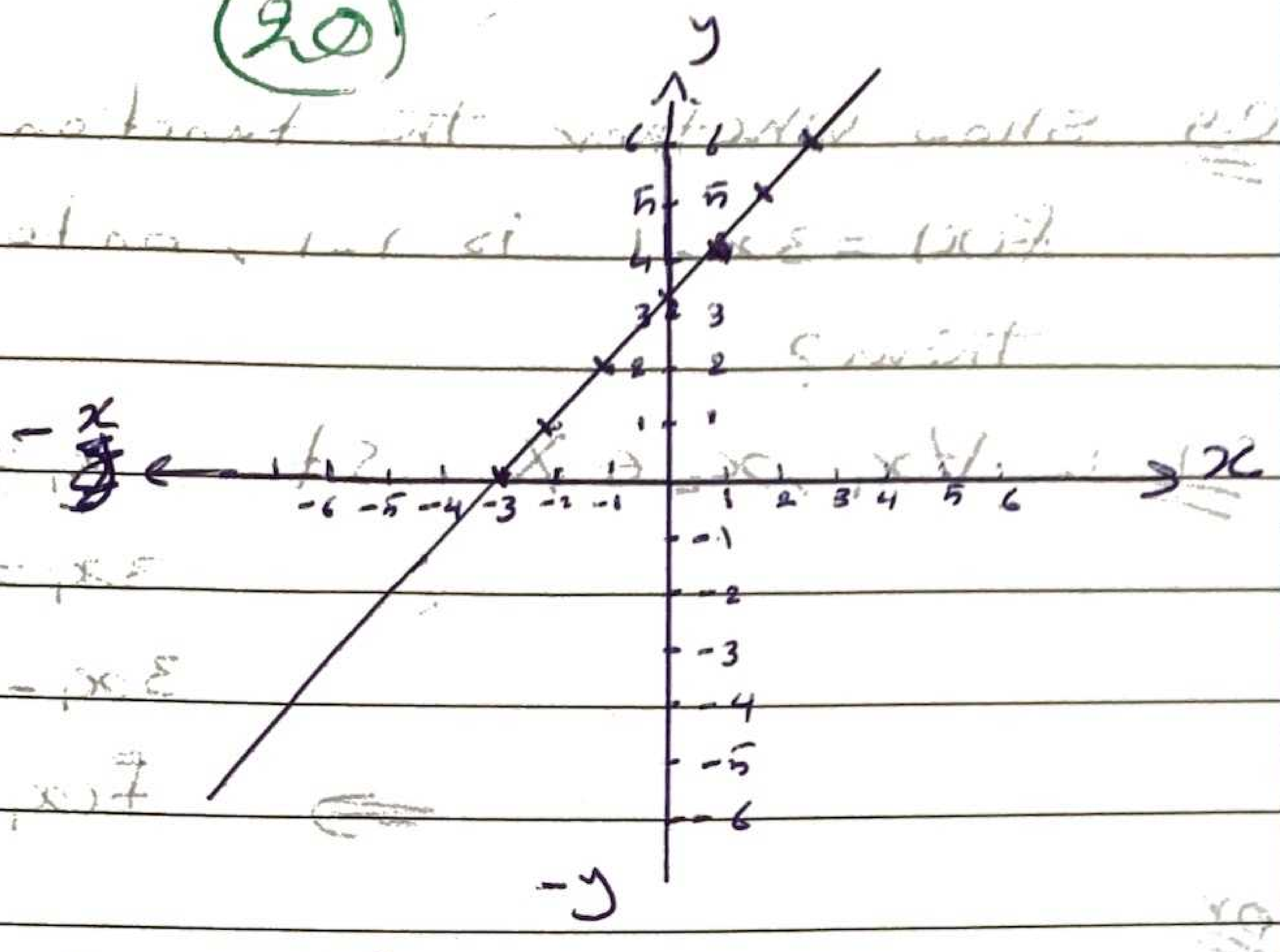
Sol

x	$y = f(x) = 3 + x$	(x, y)
-3	$f(-3) = -3 + 3 = 0$	$(-3, 0)$
-2	$f(-2) = 3 - 2 = 1$	$(-2, 1)$
-1	$f(-1) = 3 - 1 = 2$	$(-1, 2)$
0	$f(0) = 3 + 0 = 3$	$(0, 3)$
1	$f(1) = 3 + 1 = 4$	$(1, 4)$
2	$f(2) = 3 + 2 = 5$	$(2, 5)$
3	$f(3) = 3 + 3 = 6$	$(3, 6)$



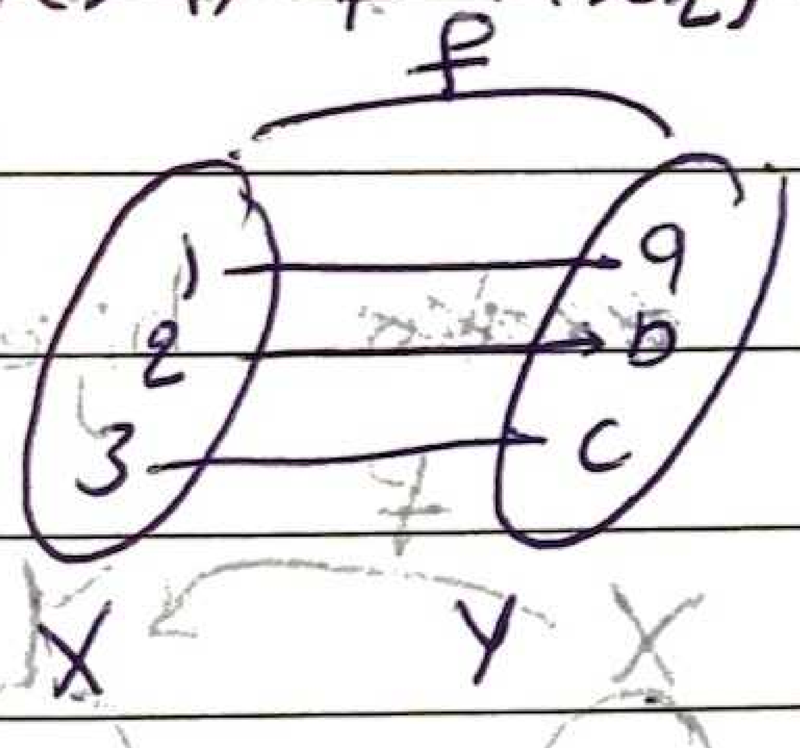
$\therefore$ the graph of f is

$$\text{graph}(f) = \{(-3,0), (-2,1), (-1,2), (0,3), (1,4), (2,5), (3,6), \dots\}$$



Some special functions:

Definition: ① A function $f: X \to Y$ is called a one-to-one (1-1), (injective) if for every two points $x_1, x_2 \in X$ s.t $x_1 \neq x_2$ then $f(x_1) \neq f(x_2)$ or if $f(x_1) = f(x_2)$ then $x_1 = x_2$



② A function $f: X \to Y$ is called onto (surjective) if for all ($\forall$) $y \in Y$, $\exists x \in X$ s.t $f(x) = y$.

③ A function $f: X \to Y$ is called bijective if it is both (1-1) and onto

e.g Show whether the function ~~[illegible]~~ $f(x) = 3x - 1$ is 1-1, onto or neither of them?

Sol: $\forall x_1, x_2 \in X$ s.t $x_1 \neq x_2$ $(* 3)$

$$3x_1 \neq 3x_2 \quad -1$$
$$3x_1 - 1 \neq 3x_2 - 1$$
$$\Longrightarrow f(x_1) \neq f(x_2)$$

or

$\forall x_1, x_2 \in X$ s.t $f(x_1) = f(x_2)$

$$3x_1 - 1 = 3x_2 - 1 \quad (\text{add}(1))$$
$$3x_1 = 3x_2 \quad (\div 3)$$
$$x_1 = x_2$$

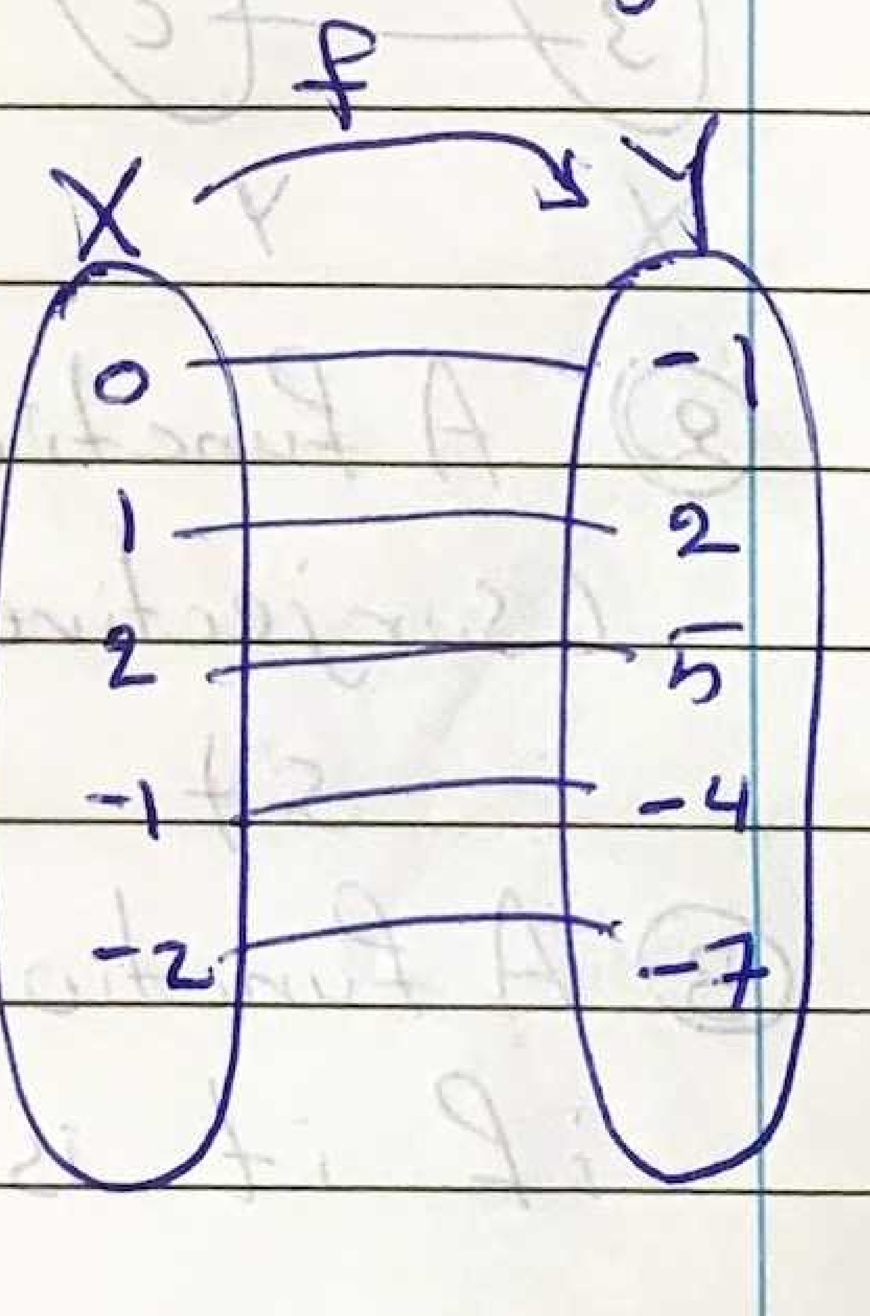
So the function f is 1-1

Since $\forall y \in Y$, $\exists x \in X$ s.t $f(x) = 3x - 1$

So the function f is onto

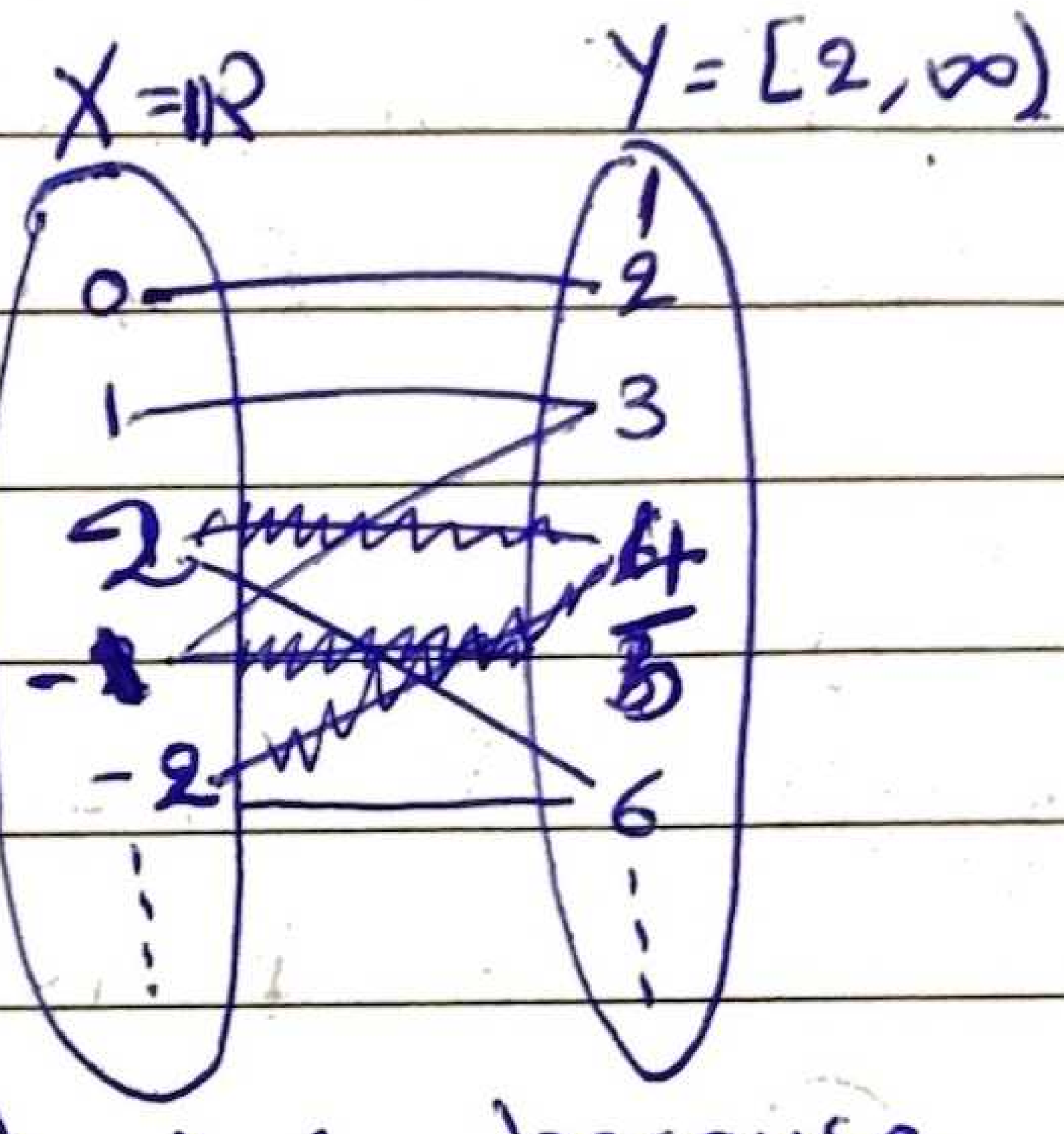
Therefore, the function f is ~~[illegible]~~ bijective

x	$y = f(x) = 3x - 1$
0	$f(0) = 3(0) - 1 = -1$
1	$f(1) = 3(1) - 1 = 2$
2	$f(2) = 3(2) - 1 = 5$
-1	$f(-1) = 3(-1) - 1 = -3 - 1 = -4$
-2	$f(-2) = 3(-2) - 1 = -6 - 1 = -7$



e.g $f(x) = x^2+2$, $f: \mathbb{R} \rightarrow [2, \infty)$

x	$y = f(x) = x^2+2$	
0	$f(0) = 0^2 + 2 = 2$	
1	$f(1) = (1)^2 + 2 = 3$	equal
2	$f(2) = (2)^2 + 2 = 6$	equal
-1	$f(-1) = (-1)^2 + 2 = 3$	
-2	$f(-2) = (-2)^2 + 2 = 6$	



The function f is not 1-1, because

$2 \neq -2$ but $f(2) = f(-2) = 6$

& the function f is onto because

$\forall y \in [2, \infty)$, $\exists x \in \mathbb{R}$ s.t $f(x) = x^2+2$

e.g If $y = x^2+1$ where $f: \mathbb{R} \rightarrow \mathbb{R}$

The function f is not 1-1 (as shown in the previous ~~eg~~ example).

The ~~fu~~ function f is not onto, because

e.g $-5 \in \mathbb{R}$ but there is no any point $x \in D_f$ s.t $f(x) = -5$

(4) A function $f: X \to Y$ is called an even function if $f(-x) = f(x)$, $\forall x \in X$.

(5) A function $f: X \to Y$ is called an odd function if $f(-x) = -f(x)$, $\forall x \in X$

(1) $f(x) = 2x+3$

sol (1) $f(-x) = 2(-x)+3 = -2x+3 \neq f(x)$

$\therefore$ f is not even

(2)

$f(-x) = 2(-x)+3 = -2x+3 = -(2x-3) \neq -f(x)$

$\therefore$ f is not odd

(2) $f(x) = x^2$

$f(-x) = (-x)^2 = x^2 = f(x)$, $\therefore$ f is an even function

and $f(-x) = (-x)^2 = x^2 \neq -f(x)$

$\therefore$ f is not an odd function

(3) $f(x) = x^3$

sol

$f(-x) = (-x)^3 = -x^3 \neq f(x)$

$\therefore$ f is not an even function

$f(-x) = (-x)^3 = -x^3 = -f(x)$

$\therefore$ f is an odd function

H.W (1) $f(x) = 1/x$

(2) $f(x) = 4-x^2$

Operations of function

If f and g are two functions, then

① $(f+g)(x) = f(x) + g(x)$

② $(f-g)(x) = f(x) - g(x)$

③ $(f \cdot g)(x) = f(x) \cdot g(x)$

④ $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, where $g(x) \neq 0, \forall x$

⑤ $(f \circ g)(x) = f(g(x))$

e.g $f(x) = \sqrt{x}$, $g(x) = x^2+2$

$(f \circ g)(x) = f(g(x)) = \sqrt{x^2+2}$

$(g \circ f)(x) = g(f(x)) = (\sqrt{x})^2 + 2$
$= x+2$

$(f \circ g)(x) \neq (g \circ f)x$

e.g If $f(x) = 5+2x^2$, $g(x) = 3-\cancel{4}x^2$

① $(f+g)(x) = f(x) + g(x) = 5+2x^2+3-4x^2$
$= 8-2x^2$

② $(f-g)(x) = f(x) - g(x) = 5+2x^2-(3-4x^2)$
$= 5+2x^2-3+4x^2$
$= 2+6x^2$

③ $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{5+2x^2}{3-4x^2}$